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THE

Accomptant's Oracle;

OR,

KEY to SCIENCE.

BEING A TREATISE OF

COMMON ARITHMETIC:

With the DOCTRINE of

VULGAR and DECIMAL

FRACTIONS.

UPON A PLAN ENTIRELY NEW.

To which are added Decimal Tables; with
their Uſe and Conſtruction.

By WARDHAUGH THOMPSON,
Many Years an ACCOMPTANT in *London*.

WHITEHAVEN:
Printed by *J. WARE*, for the AUTHOR.
M, DCC, LXXI.

181. g. 123.

PREFACE.

THE number of Treatiſes on this Subject, which are ſo frequently uſhered into the World, under the Sanction of great and illuſtrious Titles (on the Addition of this ſmall Tract) will very probably ſtart two great Queſtions againſt it. The 1ſt *is,* Why it makes its Appearance, without a Dedication? *and the* 2d, What Occaſion is there for its appearing at all? *To ſpin out the few formal Lines of a Preface, I gladly embrace an Opportunity of anſwering them, and declaring my Reaſons for publiſhing: for my Predeceſſors in arithmetical Writings, however they may have left uncultivated the Flowers of that Science, have left a ſmall Field for prefatory Diſcourſes. But happy for me, the limits of my Book, and the extenſive Liſt of my Subſcribers have confined all my excuſes to the Compaſs of three ſmall Pages.*

IN anſwer to the firſt Objection, I can only ſay, that if the Work has any Merit, the Public have diſcernment enough to find it out, and, I hope, will be generous enough to acknowledge it; if it has none, it would not only diſgrace its Patron, but render the Author ſtill more contemptible; and after all, (regardleſs of the Dignity of a Dedication) every Perſon undoubtedly has, and every judicious Perſon certainly will aſſert his Right of judging for himſelf.

AS to the ſecond Objection, many People may ſtumble upon it; eſpecially thoſe Gentlemen who ſeldom give themſelves the Trouble of looking further than the Title Page; for there——every Treatiſe is complete, and every rule adapted to the meaneſt Capacity; whether theſe Title-page-promiſſes have in any Degree been perform'd, I appeal to thoſe who have made Tryal of them. For my own part, I have heard many Gentlemen, whom I have had the Honour to qualify for public Offices, (and *who were not only indefatiguable in their Studies, but Men of clear Underſtanding) declare that the utmoſt improvement they could ever reap by reading any of them, was no more than a ſuperficial Knowledge of the Rules, without being able to give the leaſt Reaſon, why any particular Operation ſhould produce the Effect. Their Perſuaſions and Aſſurances that* if I would publiſh as I taught, I could not fail of Encouragement, *were my firſt Motives for printing this Treatiſe. I do not mean by this Declaration, to call in Queſtion the the Abilities of any Author; but Knowledge is one Thing, and the Art of communicating that Knowledge, is another; In this laſt Reſpect only, any Work of this Nature can be properly ſaid to be* NEW; *for the Principles muſt always continue the ſame.*

THIS leads me to ſay ſomething concerning my Plan; firſt then, I have endeavoured all thro' the Work (and I hope not in vain) to make every Rule depend (only) on what has gone before it.

2dly. *In defining any Propoſition, where I foreſaw any ſeeming Objection that might be ſtarted, which I could not anſwer by one continued Chain of reaſoning (without making Digreſſions from the principal Point) I have in that Caſe thrown it into Queſtion and Anſwer, it being not only the ſhorteſt, but alſo the moſt natural Way of introducing them; and although I have always thought Books wrote entirely in that Manner, very trifling and diſguſtful, yet I hope in all Places where I have made uſe of it, it will not only be pardoned, but approved.*

And 3dly. *In all Places where I thought there was a probability of the Learner miſtaking the Principles on which the Operation depended, I have there illucidated them, by giving both Operations at large. And* *although it may ſeem ſtrange to him, why the two different Operations ſhould not produce the ſame Effect, (there being a ſeeming Degree of Reaſon in both) yet by this means, I not only convince them of the propriety of the one, but the inconſiſtency of the other.*

THESE, I think, are the only ſignificant Articles, I have to take notice of; and for the reſt, without any other Introduction, I humbly ſubmit it to your Deciſion; after having firſt returned my moſt grateful Thanks to my Friends, for their generous Subſcriptions.

W. THOMPSON.

Whitehaven,
April 3, 1771.

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[illegible]

The Explication of ſome Marks,

Made Uſe of in this Work.

$=$ Two parallel lines are the marks of equality; $12d=1s$ That is, 12 pence are equal to 1 ſhilling.

$+$ St George's croſs ſignifies more, or Addition; as $5+3=8$. That is, 5 more 3, or 3 added to 5 is equal to 8.

$-$ A ſtreight line ſignifies leſs, or Subtraction; as $4-2=2$. That is, 4 leſs 2 is equal to 2.

$\times$ St Andrew's croſs denotes Multiplication; as $5\times3=15$. That is 5 multiplied by 3, is equal to 15.

$\div$ A line between 2 points is the ſign of Diviſion; as $15\div3=5$. That is, 15 divided by 3 is equal to 5.

$: :$ Four points, ſet in the middle of 4 numbers, denote them to be proportional to one another, by the Rule of Three, As $2 : 4 : : 8 : 16$ That is, As 2 is to 4, ſo is 8 to 16.

N. B. In ſome lines of the Decimal-Tables, the Periods which ſhould divide the Integers, do not appear: but the Diſtances betwixt thoſe and the Decimal-Parts, will be a ſufficient Diſtinction.

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OF

ARITHMETIC

In General.

THE knowledge of ARITHMETIC is not only abſolutely neceſſary as it relates to *Trade* and *Commerce*, (which cannot be carried on to any conſiderable extent without it) but it is alſo productive of further advantages; for it teaches us the art of reaſoning, enlarges our ideas, and fits and prepares the mind for ſtudies more difficult and abſtruſe.

ARITHMETIC conſiſts of two parts, *viz.*
WHOLE NUMBERS, and
FRACTIONS.

Arithmetic in Whole Numbers, otherwiſe called *Integers*, is when the numbers are entire, or whole quantities, and not divided into parts; ſo Two hundred and twenty ſive pounds, are Two hundred and twenty five whole quantities, or *Integers.*

BUT if we were to write down Two hundred and twenty five pounds, five ſhillings; then the Two hundred and twenty five pounds would be the *Integers*, as before; and the five ſhillings would be a broken or fractional part of one of thoſe *Integers*, or (which is the ſame thing) part of one pound.

AGAIN, were we to expreſs Eighteen ſhillings and ten pence two farthings; then the Eighteen ſhillings would be deemed *Integers*, and the Ten pence two farthings would be a fractional part, or parts, of one ſhilling or *Integer*: ſo you may obſerve, that the higheſt denomination, or name of the thing given (whether of money, weight, or meaſure) is always to be eſteemed the *Integer* or *Integers*; and the leſſer denominations, or things of leſſer value, are parts or fractions of one of thoſe *Integers.*

THE nature and uſe of all arithmetical operations is, that by ſome quantities which are given, we find out others which are required.

AND here you are to note, that notwithſtanding in moſt caſes where any of the given terms are of divers denominations, ſuch as *pounds*, *ſhillings* and *pence*, or *hundred-weights*, *quarters* and *pound weights*, &c. I ſay altho' in moſt of thoſe caſes, the method of computing by fractions may be both much ſhorter, and more exact; yet by the help of REDUCTION, which I ſhall ſhortly have occaſion to treat of, moſt queſtions in common affairs may be performed by whole numbers; which altho' they may not always give the true anſwer, yet you may come ſo very near it, that the difference will be only ſome fractional part or parts of of the loweſt name or denomination (whether of money, weight or meaſure) into which either of them can be divided; as part of a farthing, part of a pint, or part of a grain, &c. which are all ſo very inconſiderable, as in moſt calculations not to be regarded.

THE firſt part of this work therefore treats of arithmetic in whole numbers, and which (as is generally underſtood) includes *Integers* of whatever ſort, or kind, with their ſeveral known parts or denominations into which they are capable of being divided.

THE doctrine of *Vulgar* and *Decimal* fractions, with their particular uſe and excellence, I ſhall treat of in the ſecond part of this work, by which time the reader will be better prepared for a more particular definition of them.

THE four fundamental rules by which every arithmetical operation is performed, are, *Addition*, *Subtraction*, *Multiplication* and *Diviſion*; which although very ſimple and eaſy (of themſelves) to be underſtood, yet on the right management of thoſe four rules, and knowing when to uſe any one or other of them, conſiſts the whole art of *Arithmetic*.

TO theſe four rules ſome add a fifth, which is called Notation or Numeration; but as it cannot be ſaid to afford us the leaſt help in any part of the operation,

ration, ſo I think it cannot properly be called a rule; however as it is abſolutely neceſſary to be underſtood before we enter on any of the reſt; I ſhall in the firſt place ſhew what it is, and its uſe.

CHAP. I.

Of Numeration, Addition, Subtraction, Multiplication and Diviſion.

NOTATION OR NUMERATION.

NUMERATION teaches us to know the different value of figures by their different places, and of conſequence to read or write any ſum or number propoſed, which is performed by thoſe ten characters, or figures following, viz.

1,	2,	3,	4,	5,	6,	7,	8,	9,	0.
One	Two	Three	Four	Five	Six	Seven	Eight	Nine	Nought

AND by the various combinations and repititions of thoſe ten figures, all manner of numbers, which can be conceived, may be expreſſed from the leaſt to the the greateſt, even without end; but as nine figures are ſufficient for moſt ordinary concerns, ſo the table of numeration extends no farther than to nine places.

TABLE

TABLE I.

1	Units
10	Tens
100	Hundreds
1000	Thouſands
10000	Tens of thouſands
100000	Hundreds of thouſands
1000000	Millions
10000000	Tens of millions
100000000	Hundreds of millions

FROM the above table may be obſerved that every figure hath two values, viz. one in itſelf, and the other from the place it ſtands in.

THE uppermoſt figure being the firſt place, or place of units, is but ſimply one; but being removed one place furher towards the left hand, which is the ſecond place, or place of tens, it becomes Ten; again, moving it yet one place further towards the left hand, it becomes one hundred, and ſo on, increaſing ſtill in a tenfold proportion towards the left, every unit in the figure towards the left, being ten times the value of an unit in that figure which ſtands next on the right; the order of places proceeding from the right hand towards the left, but are to be read from the left hand towards the right.

TABLE

TABLE II.

Millions			Thouſands			Hundreds			
Hundreds of millions	Tens of millions	Millions	Hund. of thou.	Tens of thou.	Thouſands	Hund.	Tens	Units	
								3.	Firſt place
							8	3.	Second place
						5	8	3.	Third place
					3.	5	8	3.	Fourth place
				8	3.	5	8	3.	Fifth place
			5	8	3.	5	8	3.	Sixth place
		3.	5	8	3.	5	8	3.	Seventh place
	8	3.	5	8	3.	5	8	3.	Eighth place
5	8	3.	5	8	3.	5	8	3.	Ninth place

FOR the eaſier underſtanding how to read or write any number, I have divided the 2d table into three equal periods, and made each period to conſiſt of the ſame figures; by which you may perceive, that every three figures have the ſame return of the value of places; with this difference only, that the firſt figure, in the firſt period towards the right hand (which is the place of units) ſtands but for ſo many ſimple units as the figure expreſſes; the ſecond figure, as ſo many tens, and the third, ſo many hundreds: whereas the firſt figure in the 2d period, which is the place of thouſands, and which may properly be called units place of thouſands, ſignifies ſo many thouſands as there are units contained in the figure; the ſecond figure ſo many tens of thouſands, and ſo of all the reſt,

HENCE you ſee that every period will be read five hundred and eighty three, only you are to obſerve, that the firſt period is ſimply five hundred and eighty three, the 2d period five hundred and eighty three thouſand, and the 3d, five hundred and eighty three million.

TO know therefore the value of any number, begin at the firſt figure towards your right hand, calling it units, the 2d tens, and the 3d hundreds; making a point or comma every third figure, by which means you will find no difficulty in reading any number of figures under ten, provided that you have made yourſelf thoroughly maſter of the value of any three figures in all the various ſituations in which they can poſſibly be placed.

LET us for example take the following three, viz. 500; theſe as they now ſtand ſignify five hundred; but place them thus, 050 they ſignify but fifty, and again if we place them in this order, 005, they amount to no more than five.

LET us for another example take two ſignificant figures, with one nought or cypher, which may be 580, theſe as they now ſtand are Five hundred and eighty, place them thus, 508, they make five hundred and eight, and placed thus 058, only fifty eight.

THIS laſt example would ſtill admit of a greater variety of changes, which conſequently would alter the value of them; but as the placing any one of the ſignificant figures before or after another, could not make them more difficult to be read; the different manners in which we have ſhewed them here, is therefore ſufficient for our purpoſe.

THOSE tables and inſtructions well underſtood, I think the learner can be at no leſs to write down any number whatſoever; but as I have ſeen many perſons who could read any number when it was wrote, and yet could not write down the ſame when propoſed to them; I ſhall here add a third table which will ſtill further aſſiſt them.

TABLE III.

Hundreds of millions	Tens of millions	Millions	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units	
								1	Firſt place
							3	0	Second plac
						5	0	0	Third place
					0	0	0	0	Fourth place
				7	0	0	0	0	Fifth place
			0	0	0	0	0	0	Sixth place
		6,	0	0	0	0	0	0	Seventh place
	8	0,	0	0	0	0	0	0	Eighth place
9	8	0,	0	0	0,	0	0	0	Ninth place
9	8	6,	0	7	0,	5	3	1	
Millions			Thouſands			Hundreds			

NOW ſuppoſe it were required to write down in figures, Five hundred and thirty one: here you are to take notice, that it will not take the ſame quantity of figures to expreſs that number together, as it doth to expreſs them ſeverally, according to the table; for then it would ſtand thus 500301; which according to the rules already given to find the value of places, 3 would ſtand in the place of hundreds, which ought to have ſtood in the place of tens; and 5 in the place of hundreds of thouſands, which ſhould only have been in the place of hundreds; ſo that the number thus ſet down would be Five hundred thouſand three hundred and one.

WHEN therefore any number is propoſed to be wrote down in figures, as, for example, Nine hundred and eighty ſix million, ſeventy thouſand, five hundred, and thirty one, which is the ſame with that number in figures under the black line below table 3d.

FIRST then enquire what was the laſt number mentioned, which you will find to be one, and becauſe you ſee by table 3d that it ſtands in the firſt place, or place of units, you therefore ſet it down the firſt figure towards the right hand, as you ſee in the line of figures below table 3d.

AGAIN examine what is the next number wanted, which you will find to be Thirty, and which you ſee by the table takes up two places, viz. the place of units and the place of tens, but becauſe the place of units is already ſupplied by your 1, ſo you have only to place your 3 to the left hand of it, viz. the place of tens, and then you have 31, as you ſee.

THE next number wanted is five hundred, which according to the table takes up three places of itſelf, but the place of units and tens being already filled up, you therefore place your five to the left hand of your three, which will then poſſeſs the place of hundreds, and compleat your firſt period of 531. You next proceed to fill up your 2d period, and finding the number wanting for it to be ſeventy thouſand, you therefore ſupply the units place of thouſands with a nought or cypher, in order that your ſeven may be removed one place further towards the left hand, which is the 5th place, or place of tens of thouſands; and although you have no hundreds of thouſands, yet you muſt alſo ſupply that place with a cypher to compleat your 2d period, as the number propoſed extended beyond it, otherwiſe there would have been no occaſion to have placed a cypher to the left of your ſeven, as cyphers to the left hand of any figure neither add to, or take from the value of them, and are therefore of no uſe except when there is a ſignificant figure to be placed to the left hand of them, as in the preſent caſe, where had you placed your 6 (which is the next figure wanted) to the left of your 7 without a cypher betwixt them, then your 6 would have come within your 2d period, and conſequently been but ſix hundred thouſand, whereas you ſee it is the firſt place of your 3d period and therefore ſix million; and ſo of all the reſt.

HAVING now, I hope, ſufficiently explained the numeration table, and ſhewed you how any number may be wrote down, I ſhall here propoſe ſome few for the exerciſe of the learner.

Write down in figures the following numbers.

Eighty ſeven.
Three hundred and four.
Fifty two thouſand and fifty ſix.
Four hundred and nine thouſand, and five.
Six million, four hundred, and twenty eight.
Ten million, and twenty four thouſand,
Five hundred million, ſeven hundred and five.
Six hundred and forty nine million, ſeven hundred and twenty eight thouſand, eight hundred and ſixty four.

ADDITION.

ADDITION teaches us to bring any two or more numbers into one ſum, and is either ſimple or compound.

Simple addition is the adding of ſeveral whole quantities or Integers of the ſame kind or denomination together, as 12 yards, 15 yards, and 25 yards, which being added together make in all 52 yards, which is called the Total Sum.

WHEN ſeveral numbers of the ſame denomination are given to be added together, you muſt carefully obſerve to place units under units, tens under tens, and hundreds under hundreds, &c.

FOR example, were it required to add 5

16 pounds, and 105 pounds together; firſt ſet your 5 in the place of units, your 16 next ſet in this manner, viz. Your 6 being likewiſe ſix units, you place it directly under your 5, and the 1 which is one ten, muſt be placed to the left hand of your 6, which is the 2d place or place of tens; you muſt place your 105 as follows, viz. your 5 in the units place, which will be directly below your 6; your cypher under your one, which is the place of tens; and then of courſe your 1 being placed to the left hand of your cypher will poſſeſs the place of hundreds; and then they will ſtand as under.

$$\begin{array}{r}5\\16\\105\\\hline 126\end{array}\quad\text{or thus}\quad\begin{array}{r}5\\105\\16\\\hline 126\end{array}\quad\text{or thus}\quad\begin{array}{r}16\\5\\105\\\hline 126\end{array}$$

$$\text{or thus}\quad\begin{array}{r}16\\105\\5\\\hline 126\end{array}\quad\text{or thus}\quad\begin{array}{r}105\\5\\16\\\hline 126\end{array}\quad\text{or thus}\quad\begin{array}{r}105\\16\\5\\\hline 126\end{array}$$

WHICH are all the various ſituations in which they can be properly placed; and although every one of the above examples differ one from another, with reſpect to the order of places upwards or downwards; yet you ſee the order of places from right to left (and from which they derive their ſecond value) is ſtill the ſame.

THE firſt and laſt of thoſe examples are however the moſt natural way of placing them, though all of them anſwer the ſame purpoſe.

BUT that I may omit nothing which might make me more perfectly underſtood, I ſhall here ſhew the learner two or three different ways in which they would have been wronged placed.

5 . .	. 5 .	105 .
16 .	16	15
105	105	5
756	171	1070

AS the firſt of the above examples is the moſt probable error into which the learner might fall, I ſhall only explain it, which I hope will ſufficiently prove the impropriety of the whole.

AND here note, that for the more eaſy diſcovering the value of places, I have made a dot thus (.) to ſupply the place of a cypher, or cyphers.

HENCE you will clearly perceive that the 105 is the only number which preſerves its true value or place; for ſuppoſing the dots to be cyphers, you will find that the 5 in the upper line, which in their right order ſtood in the place of units, (and conſequently but ſimply 5 units or ones) now becomes 500; and the 16 which only filled the place of units and tens, is now removed to the place of tens and hundreds; ſo that inſtead of 16, it becomes 160.

THAT the learner may therefore be thoroughly convinced of what conſequence it is to place numbers in their proper order, I have added up the ſeveral examples as they ſtand right, and wrong placed; by which he may ſee, that notwithſtanding the many different ſituations in which they ſtand right, yet the ſum total is ſtill the ſame; whereas thoſe wrong placed, not only differ one from another, but all of them widely from truth.

SUPPOSING the learner now well acquainted with the manner of placing his numbers, I ſhall next give him a few examples for practice.

EXAMPLES.

1ſt.	2d.	3d.	4th.	5th.	6th.
£	*Yds*	*Days*	*Tons*	*℔*	*Feet*
7	47	301	3270	27013	270864
6	84	127	4735	56427	302153
3	26	586	6824	32104	623081
8	35	234	5821	87052	437657
2	79	787	7430	63279	321040
9	68	625	6785	33796	562372
4	25	154	2573	25370	437809
3	21	359	1042	72321	786425
42	385	3173			

Ex. 7 There are 6 purſes each containing 2358 pounds; I demand how many pounds there were in all.

Ex. 8 There are 6 caſks of rice, each containing 3783 pound; I demand how many pounds weight were in the whole.

£	lb.
2358	3783
2358	3783
2358	3783
2358	3783
2358	3783
2358	3783
14148	

Scho. I obſerve in the foregoing examples (whether of money, weight, &c.) that you ſtill carry at tens, and ſhould be glad to know the reaſon.

Maſt. THE explanation already given of your numeration table, fully anſwers that queſtion; but if further inſtructions be neceſſary, obſerve the following.

IN all whole quantities or Integers (of which are the foregoing examples) the value of places from the right hand to the left, increaſe in a ten-fold proportion, as has been ſhewn before; every unit in the figure towards the left hand, being ten times the value of an unit in that figure which ſtands next it on the right. The 1ſt line is therefore a line of units, the 2d line all tens, and the 3d line all hundreds, &c.

WHEN therefore you have added up your firſt line, as in the 3d example, which comes to 43, ſet down the 3 units directly under the line, and the 4 tens which are contained in it you carry or add to the 2d line which (as I ſaid before) is likewiſe all tens; then next adding your 2d line up which comes to 37; and which is 37 tens; and ten tens, or ten times ten, being a hundred, you therefore find it amounts to three hundred, and 7 tens over; ſo ſetting down the 7 odd tens under the 2d line, or line of tens, you carry your 3 (which are three hundreds) to the next line, being the line of hundreds; and adding it up as before, find the amount to be 31, which is 31 hundreds; and as ten hundred is a thouſand, conſequently it will be 3 thouſand and 1 hundred over, which 1 you place below the 3d line, or line of hundreds; and becauſe there are no more lines to the left hand to carry or add the 3 thouſand to, you therefore ſet it down to the left hand of your 1, and the work appears as you ſee.

Of COMPOUND ADDITION.

COmpound Addition is the adding ſeveral numbers of the ſame kind together, although of different denominations, as pounds, ſhillings and pence; hundred weights, quarters, and pounds, &c.

SECTION 1ſt.

WHEN ſeveral numbers of divers denominations are given to be added together; (whether of money, weight or meaſure) firſt begin with the line towards the right hand, and conſider how many of that denomination are equal to one of the next denomination, towards the left (which you will ſee by your table) and ſo many times as you find that number contained in the ſum of your firſt line, juſt ſo many muſt you add, or carry to your next line towards the left: again ſee how many of that denomination of your 2d line are equal to one in the next on the left hand; and as often as that number is found, ſo many you muſt carry to the next, and ſo proceed in the ſame manner until you come to the laſt denomination, at which you muſt always carry at tens.

Of *Money*.

4	Farthings make	one Penny, mark'd thus	*d.*
12	Pence - - - -	one ſhilling, - - - - -	*s.*
20	Shillings - - -	one pound *ſterling*, - -	£

THERE are alſo ſeveral pieces of money current in England, which that the reader may be acquainted with, I ſhall here ſubjoin a table of their value.

PORTUGAL MONEY.

	£	*s*	*d*
One piece	=3	12	0
A 36 ſhilling piece	=1	16	0
A half d°	=1	18	0
Quarter d°	=0	9	0
An eighth d°	=0	4	6
A moidore	=1	7	0
A half moidore	=0	13	6
A quarter-moidore	=0	6	9

ENGLISH MONEY.

	£	*s*	*d*
A guinea	=1	1	0
Half d°	=0	10	6
Quarter d°	=0	5	3
A Crown	=0	5	0
A half crown	=0	2	6

THE following pieces were formerly in uſe, but are now only imaginary; as we have no particular pieces of coin to expreſs their reſpective value.

	£	s	d		£	s	d
A Carolus =	1	3	0	An Angel =	0	10	0
A Jacobus =	1	5	0	A Noble =	0	6	8
A Mark =	0	13	4				

BEFORE I proceed to give you any examples in Addition of Money, I muſt firſt obſerve to you, that it is cuſtomary to write down farthings fractionally, which way (however) is very improper in this place, before the learner can be ſuppoſed to be acquainted even with whole numbers: and indeed at any rate, altogether unneceſſary, ſince they might be expreſſed ſimply, as well as any other of the leaſt denominations of weight, meaſure, &c. But as the other method of writing them is generally practiſed, I ſhall here ſhew how they are to be expreſſed.

One Farthing wrote thus	¼
Two D° thus	½
Three D° thus	¾

EXAMPLES.

1ſt.			2d.			3d.		
£.	s.	d.	£.	s.	d.	£.	s.	d.
4	8	6	5	10	7 ¾	12	17	9
2	7	4	7	12	9	15	14	11¾
5	9	10	4	9	6 ½	24	16	8¼
7	4	4	8	13	4 ½	36	10	6¾
8	7	8	9	15	7 ½	48	9	8¼
27	17	8	36	1	11			

5th.			6th.		
£.	s.	d.	£.	s.	d.
4123	15	4¼	2743	10	6¾
5301	12	5½	1922	9	11½
2123	10	8	3057	15	9¾
7234	8	4¾	7125	12	10¼
8321	19	7¾	3204	16	4¾
3105	15	8½	8357	14	8¾
8713	12	10¾	4279	15	7½
5875	10	11½	3457	10	10¼

TO add up any of the foregoing examples, as for inſtance, the firſt; begin with the line which ſtands laſt, towards the right hand, which is pence; and becauſe 12 pence are equal to one of the next denomination, viz. One ſhilling; you therefore at every 12 make a dot, thus (.), adding ſtill the over-plus, if any, to the next figure, and when the line is finiſhed, then if there be any thing over 12, ſet it down under the line of pence; and ſo many dots or points as you have, are juſt ſo many ſhillings, which you carry to the line of ſhillings; and becauſe 20 ſhillings make one pound, you therefore in adding up this line, make a dot at every twenty, carrying the over-plus to the next figure, as before directed; and when the line is finiſhed, ſet the odd ſhillings, if any there be, down under your line of ſhillings, and carry ſo many dots as you have, to your next line, which being the laſt denomination towards your left hand, and conſequently the higheſt, are therefore eſteemed whole quantities, or integers, at which you are always to carry at tens, as directed in *Addition of whole numbers.*

THERE is another method which ſome maſters uſe inſtead of pointing, (but with reſpect to money only) and that is, to give the learner a table to be got by heart, ſhewing how many ſhillings are contained in any number of pence, not exceeding 120. But ſince this table is too limited to be of general uſe, I did not think it worth inſerting here.

S. But might not ſuch a table be extended beyond 120?

M. Yes: but as you will by your Multiplication table (when learned) more readily find how many ſhillings are contained in any number of pence not exceeding 144; and by Diviſion, how many ſhillings there are in any number of pence whatſoever; I would therefore adviſe you, till you have learned thoſe, to uſe the method of pointing; which although it would have a very blundering appearance in a man of buſineſs, yet in this place is not only very allowable, but alſo very proper; ſince it would be highly inconſiſtent with common reaſon, (on which the plan of this work is founded) to propoſe working the preſent rules, by any method which depends on the knowledge of thoſe with which we ſuppoſe the learner yet unacquainted.

THE uſe of your Mulplication Table in the abovementioned caſes, will naturally ſuggeſt itſelf to you: and when you have got as far as Diviſion, I ſhall give you ſome examples how to apply it: which will anſwer in all caſes of Addition, whether of money, weight, or meaſure, &c.

THE following little practical Queſtions, may ſerve to ſhew the uſe of Addition, as alſo the forms of drawing out bills of parcels, book debts, &c.

A MERCER'S Bill of Parcels.

Mr. JAMES TODD,

Bought of ROBERT SCOTT.

January 3d, 1771.	£.	s.	d.
6 Yards of Silk at 7*s*. 6*d*. per yard	2	5	0
9 Yards of black ſattin, at 12*s*.	5	8	0
4 d° of flowered d° at 14*s*.	2	16	0
2 d° of Genoa velvet at 18*s*.	1	16	0
[illegible] d° of ribbon at 8*d*.	8	5	4
Total Sum			

A Woolen Draper's Bill.

Mr. JOHN GRAY, To SAMUEL STEVENSON Dr.

			£	s	d
January	5	To $3\frac{1}{2}$ yards of ſuperfine cloth, at			
1771.		18*s*. per yard	3	3	0
		To $5\frac{1}{2}$ d° ſhalloon, at 1*s*. 8*d*.	0	9	2
	25	To 8 d° of ſcarlet frieze at 10*s*. 6*d*.	4	4	0
February	7	To 12 d° of druggit at 7*s*. 4*d*.	4	8	0
March	20	To 9 d° of ſerge at 2*s*.	0	18	0
	25	To 5 d° of ſuperfine black cloth at 17*s*. 6*d*.	4	7	6
		Total Sum			

S. Pray do thoſe two bills, differ in any thing elſe than form?

M. The firſt is commonly called a bill of parcels;

and is generally delivered to the buyer at the ſame time with the goods; whether they pay ready money, or take them on credit.

THE 2d is the form in which book debts are generally drawn out; I mean goods which have been formerly ſold on credit, and for which the buyer ſtands debited in the ſellers books.

THE ſeveral ſums in the following bill, are expreſſed as you ſee, on purpoſe, to try if the learner can write them down properly in figures.

A Bill of Diſburſements.

Laid out in beef, eight groats and two pence.
In lamb, nineteen pence halfpenny.
In bread, three and twenty pence.
In veal, nine groats and a penny.
In greens, three half pence.
In candles, eighteen pence farthing.
In muſtard, two pence three farthings.

Suppoſe I am indebted

To A Two hundred and forty pounds, three ſhillings.
B Eighty four pounds, twelve ſhillings and ſix-pence.
C Five pounds, and eight pence.
D Fifteen pounds, nine ſhilling and two-pence.
E Seventeen ſhillings and ten-pence farthing.
F One pound ſeven ſhillings and three farthings.
What is the ſum?

2d Of *Troy* Weight.

24 Grains or *gr.*	make	1 Penny weight. *dw.*
20 Pennyweights	make	1 Ounce. *oz.*
12 Ounces	make	1 Pound. *lb.*

By this weight gold, ſilver, and jewels are weighed.

EXAMPLES.

1ſt.			2d.			3d.				4th.			
oz	*dw*	*gr*	*oz*	*dw*	*gr*	*lb*	*oz*	*dw*	*gr*	*lb*	*oz*	*dw*	*gr*
8	16	12	6	19	20	4	10	19	20	19	11	18	22
6	12	19	9	16	19	5	10	13	10	16	10	16	15
9	13	4	5	13	10	6	9	14	16	19	9	12	13
5	9	15	8	12	13	5	8	12	13	14	6	13	16
3	15	13	6	10	15	9	6	14	12	13	8	15	13
5	18	16	8	16	12	12	7	13	17	16	9	10	12
40	6	7											

Of *Avoirdupois* Weight.

16 Drams	make	1 ounce—*oz.*
16 Ounces	make	1 pound—*lb.*
28 Pounds	make	1 quarter of *Cwt.*
4 Quarters of a hund.	make	1 hund. weight *C.*
20 Hundred weight	make	1 Ton—*T*

By this Weight is weighed all ſorts of *Grocery* and *Chandler-wares*; and likewiſe all *Metals*, but *Gold* and *Silver*.

EXAM-

EXAMPLES.

1ſt.			2d.			3d.				
lb	*oz*	*dr*	*Cwt*	*qr*	*lb*	*Ton*	*C*	*qr*	*lb*	*oz*
12	10	14	21	2	18	36	19	3	27	10
13	12	10	14	1	6	44	17	1	21	9
17	8	9	17	2	12	18	10	0	24	8
16	15	14	12	0	21	25	15	3	21	15
18	13	12	13	3	25	37	13	2	20	13
10	11	15	19	1	27	53	9	1	26	12
17	6	13	10	2	18	21	10	2	25	9
107	15	7								

Of *Apothecaries* Weight.

20 Grains, or *gr*, make	1 ſcruple--*ſc*
3 Scruples	1 dram--*dr*.
8 Drams	1 ounce--*oz*.
12 Ounces	1 pound--*lb*.

By this Weight *Apothecaries* compound their *Medicines*.

EXAMPLES.

1ſt.				2d.				3d.				
lb	*oz*	*dr*	*ſc*	*lb*	*oz*	*dr*	*ſc*	*lb*	*oz*	*d.*	*ſc*	*gr*
6	3	6	2	7	3	7	2	9	7	2	1	11
5	2	5	1	8	2	6	1	6	5	3	2	12
7	1	4	0	9	1	5	0	9	6	4	0	13
3	11	7	2	5	3	2	1	3	3	5	1	13
2	1	3	1	6	3	5	2	3	5	2	0	14
1	7	6	1	9	1	6	0	2	4	1	2	13
5	10	7	2	5	3	4	1	9	6	3	1	10
6	4	3	0	6	2	3	2	8	5	4	0	12
38	8	4	0									

By

Of *Long* Meaſure.

3	Barley-corns, or *b. c.* make	1	Inch--*In.*
4	Inches	1	Hand--*Ha--d*
12	Inches	1	Foot---*Ft.*
3	Feet	1	Yard---*Yd.*
6	Feet	1	Fathom-*Fa.*
5	Yards and a half	1	Rod, pole or perch.
40	Poles or 220 Yards	1	Furlong--*Fu.*
8	Furlongs	1	Mile---*Mi.*
3	Miles	1	League--*L.*
60	Miles	1	Degree---*Deg.*

Note 1ſt. A degree is 69 ſuch miles, and 4 furlongs, very near; though commonly reckoned but 60 miles.

Long meaſure is uſed to meaſure the diſtance of places, or any thing elſe, where length is conſidered, without regard to breadth.

E X A M P L E S.

1ſt.			2d.			3d.				4th.			
M	*ſ*	*p*	*Yds*	*ſ*	*in*	*Le*	*m*	*ſ*	*p*	*yds*	*ſ*	*in*	*ba*
19	6	19	16	1	9	12	2	6	31	12	2	11	2
16	5	13	31	2	10	13	0	5	32	13	1	10	1
12	3	10	36	1	9	16	1	3	21	16	0	6	0
15	2	18	20	0	8	18	2	6	20	18	2	5	2
18	3	11	63	2	6	19	0	5	10	10	1	6	1
10	5	10	85	1	3	13	1	6	5	25	0	4	0
19	3	19	30	0	5	16	2	5	26	15	2	10	2
112	5	20											

Of *Cloth* Meaſure.

2 Inches and a quarter make 1 Nail
4 Nails make a quarter of a yard = 9 Inches.
4 Quarters = 1 Yard
3 Quarters of a yard make 1 Flemiſh Ell.
5 Quarters of a yard make 1 Engliſh Ell.

EXAMPLES.

1ſt.			2d.			3d.			4th.		
Yds	*qr*	*na*	*Ells*	*qr*	*na*	*E.f.*	*qr*	*na*	*Ells*	*qr*	*na*
19	3	3	20	4	3	16	1	3	20	4	3
15	2	2	10	2	0	13	1	0	19	3	2
16	1	0	15	3	2	12	2	2	16	1	1
20	3	3	19	2	3	13	0	1	10	2	2
25	2	1	12	3	2	12	1	3	16	0	1
26	3	2	13	1	1	18	0	1	19	1	3
12	0	1	36	2	3	10	1	2	11	0	1
13	2	3	15	4	2	19	2	3	16	2	0
150	3	3									

Of *Land* Meaſure.

9 Square feet	make	1 Yard.
30 Yards		1 Pole.
40 Poles in length and 1 in breadth		1 Rod.
4 Roods		1 Acre.

The uſe of Land meaſure is to give the contents of any piece of ground in acres, &c.

EX

EXAMPLES.

1ſt			2d			3d			4th		
A	*r*	*p*	*A*	*r*	*p*	*A*	*r*	*p*	*A*	*r*	*p*
18	3	31	14	3	23	14	2	36	16	2	36
16	2	30	12	3	32	10	1	12	12	2	22
19	1	20	10	2	11	12	0	13	14	0	19
16	2	10	16	1	10	16	2	13	16	2	20
12	0	12	13	3	22	13	1	10	13	1	16
14	1	13	14	0	21	12	3	13	12	0	18
16	2	11	10	2	13	15	1	22	20	1	13
114	2	7									

Of *Liquid* Meaſure.

THERE are two ſorts of liquid meaſure, viz. Wine meaſure, and Wincheſter meaſure.

A gallon of wine is 231 ſolid inches, but a gallon of beer or ale, exceeds that meaſure by 51 inches, and is 282 ſolid inches.

Of *Wine* Meaſure.

2 Pints make	1 Quart
4 Quarts	1 Gallon
10 Gallons	1 Anker of brandy or rum
18 Gallons	1 Runlet
31 ½ Gallons	1 Barrel
42 Gallons	1 Tierce
63 Gallons	1 Hogſhead
84 Gallons	1 Puncheon
2 Hogſheads	1 Pipe or Butt
2 Pipes or 4 hogſheads	1 Tun.

This meaſure is commonly uſed in meaſuring all brandies, ſpirit, ſtrong waters, perry, cyder, vinegar, honey and oil.

EXAMPLES.

1ſt.				2d.				3d.		
T	*hds*	*gls*	*qts*	*T*	*hds*	*gls*	*qts*	*Tier*	*gls*	*qts*
9	3	41	3	20	3	33	3	36	30	2
6	2	30	2	36	2	16	2	52	10	1
8	1	20	1	53	1	26	0	13	12	3
5	0	16	0	65	0	31	2	51	20	0
7	3	10	3	33	3	14	1	35	16	1
3	2	13	2	60	1	10	0	25	12	3
41	1	6	3							

Of *Wincheſter* Meaſure.

2 Pints make	1 Quart
4 Quarts	1 Gallon
8 Gallons	1 Firkin of ale
9 Gallons	1 Firkin of beer
2 Firkins	1 Kilderkin
4 Firkins	1 Barrel

1 Barrel and a half, or 54 Gallons, 1 Hhd. of beer

Note 2d. In all other parts of England, except London, the meaſure for ale, ſtrong beer, and ſmall beer, is the ſame; which is 34 gallons to the barrel, and 8 gallons and a half to the firkin.

EXAMPLES.

1ſt.			2d.			3d.		
Hhds	*gall*	*qrts*	*B.B.*	*fir*	*gall*	*A.B.*	*fir*	*gal*
9	30	3	20	2	8	26	3	7
6	21	1	35	3	6	25	2	5
8	16	0	31	1	3	21	1	8
5	12	2	21	2	5	22	3	7
3	31	1	13	0	4	20	1	5
5	10	3	12	1	3	23	0	6
6	13	2	16	3	6	27	3	4
8	12	1	13	2	5	21	2	3
52	40	1						

Of

Of Dry Meaſure.

2 Pints make	1 Quart
2 Quarts	1 Pottle
2 Pottles	1 Gallon.
2 Gallons	1 Peck
4 Pecks	1 Buſhel
8 Buſhels	1 quarter of corn
36 Buſhels	1 chaldron of coal
10 Quarters of corn	1 Wey
12 Weys.	1 Laſt.

Note 3d. In all other parts of *England*, except London, the chaldron of coals is but 32 buſhels.

EXAMPLES.

Ch	*bu*	*p*	*Qrs*	*bu*	*p*	*Qrs*	*bu*	*p.*
16	19	2	14	6	3	36	7	3
13	16	3	16	5	2	30	6	2
12	10	1	19	3	1	25	4	1
15	13	2	13	2	0	23	3	0
16	11	0	18	7	1	7	2	3
12	16	1	15	6	3	36	5	2
13	12	2	17	5	2	21	3	1
10	15	3	12	2	3	34	6	3
110	7	2						

Of *Time.*

60 Seconds make	1 minute
60 Minutes	1 hour
24 Hours	1 day
7 Days	1 Week
4 Weeks	1 Month

13 Such months, 1 day, and 6 hours, nearly make a common year.

Note, According to the beſt computations, a ſolar year is 365 days, 5 hours, 48 minutes, and 57 ſeconds.

We only reckon 365 days to the year, which according to the calendar are divided in the following manner.

Thirty Days hath September,
April, June, and November;
And all the reſt have Thirty one,
Save February which alone
Has twenty-eight; and one day more
Is added to't, one year in four.

EXAMPLES.

1ſt.			2d.			3d.			
m	*w*	*d*	*h*	*m*	*ſec*	*d*	*h*	*m*	*ſec*
26	3	6	35	26	42	14	16	26	51
25	2	5	30	36	51	16	12	21	43
36	1	4	28	25	32	12	13	22	36
31	3	3	56	30	25	19	12	27	25
13	0	6	82	21	23	19	10	21	32
33	3	5	65	37	30	12	13	23	55
16	2	2	23	51	21	16	12	21	35
11	1	2	65	23	52	19	10	30	16
195	3	5							

Of

Of Motion.

60 Seconds marked ″ make 1 Prime minute.
60 Minutes 1 Degree °
30 Degrees 1 Sign.
12 Signs, or 360 degrees, make the whole great circle of the zodiac.

EXAMPLES.

1ſt.			2d.		
°	′	″	°	′	″
14	12	13	46	50	43
69	48	41	35	21	36
35	22	16	21	32	16
12	14	10	12	13	12
20	31	18	16	14	21
16	14	10	12	10	13
22	31	13	13	11	16
16	10	12	12	13	14
207	4	13			

Queſtions to exerciſe Addition.

1ſt A Man was born in the year 1704, I demand when he will be 60 years of age.

2d There are two numbers whoſe difference is 60, and the leſſer number is 49; what is the greater number?

3d A man borrowed a ſum of money, and paid in part, 112 *l.* 10 *s.* and the remainder is 17 *l.* 10 *s.* I demand the ſum borrowed.

4th A owes me 3 guineas, B 50*l.* 12*s.* C 104 *l.* and D threeſcore and ſeventeen pounds, How much is due to me in all?

5th A man hath 6 bags of hops; the firſt weighs 2 qrs. 14lb. and each of the reſt weighs 14lb. more; what quantity had he in the whole?

6th A man took a houſe for 12 years; and by agreement was to pay 100 *l.* 10*s.* down; 190 *l.* 4 *s.* at the end of 6 years; and 109*l.* 6*s.* at the end of 12 years; I demand how much the whole ſum was?

Of SUBTRACTION.

SUBTRACTION ſhews how to take a leſſer number from a greater, ſo that the difference between them may be known.

SUBTRACTION is either ſimple or compound; but as the diſtinction is the ſame, with that which I have already ſhewn, between ſimple and compound addition; I therefore think it unneceſſary to repeat it again here: the ſame alſo is to be underſtood, both of multiplication and diviſion.

IN placing your numbers for ſubtraction: care muſt be taken, that the largeſt number be always ſet uppermoſt, and the leſſer number, regularly below it; viz. in ſimple Subtraction, (by which I mean Subtraction of whole quantities or Integers) units under units, tens under tens, &c. and in compound Subtraction, thoſe numbers under each other, which are of the ſame name or denomination.

WHEN your numbers are thus placed, begin with the laſt figure towards the right hand: which (if leſs than the figure which ſtands over it, ſubtract from the firſt figure, and the remainder, or difference, place exactly below; and in this manner proceed, ſo long as the figure in the under line, is leſs than that which ſtands over it; but when the figure in the lower line, is greater than that which it is to be taken from; you then muſt borrow one from the next figure in the upper line toward the left hand, which will always be equal to ſo many in the preſent line as you point at in addition; and then taking the lower figure out of that you borrow, add the remainder to the upper figure and ſet it down, carryng one for that you borrowed, to the next figure in the lower line towards the left hand, and ſo proceed in this manner till you have finiſhed.

EXAMPLES.

	1ſt	2d	3d	4th	5th
	£	*yds*	*days*	*months*	*crowns*
From	654	9432	5643	3210	2136
Take	321	5941	2421	2631	1328
	333	3491			

	6th	7th	8th	9th
	hours	*lb*	*ſhillings*	*miles*
From	653210	572104	921561	346452
Take	346021	256351	312156	123564

S. There are two things in the above operation, the reaſon of which I do not underſtand: The firſt is, why you borrow 10, and pay but 1 back again: and the ſecond, why you borrow from the upper line, and pay it to the under one

M. To anſwer your firſt queſtion, I need only refer you to your firſt table of numeration, which will inform you, that every unit to the left hand is equal to ten, in the next place towards the right.

And as to your 2d,

You will pleaſe to obſerve, that whether we add one to the next figure in the lower line, or ſuppoſe the next figure in the upper line (from which we borrowed it) to be one leſs than it really is, it has the ſame effect; and therefore, as it is much eaſier, and leſs burthenſome to the memory, to add one to the lower line, that method is found better for practice.

N. B. The very ſame reaſon will hold good in *Subtraction* of divers denominations, reſpect being had to the value of the one, which you borrow; I mean how many it is equal to, of that denomination to which you borrow it, as you will find more fully explained in the firſt Section of this chapter.

Of COMPOUND SUBTRACTION.

1ſt Of MONEY.

EXAMPLES.

	1ſt.			2d.			3d.		
	£	s	d	£	s	d	£	s	d
From	345	16	8	633	10	6	630	13	4
Take	231	12	6	242	12	9	210	13	2
	114	4	2	490	17	9			

	4th			5th			6th		
	£	s	d	£	s	d	£	s	d
From	360	14	6¾	169	12	3½	120	13	4¼
	120	16	5¾	96	10	6¼	60	14	6½

	7th		
	£	s	d
Borrowed	690	12	6
Paid at ſeveral times.	12	16	8
	9	8	6
	15	17	8
	25	14	0
	38	6	10
	45	17	4
Paid in all	148	1	0
Remains unpaid			

	8th		
	£	s	d
Lent	540	0	0
Received at ſeveral times	50	16	8
	25	12	6
	34	10	4
	45	16	10
	25	12	0
	15	16	8
Recd. in all	198	5	0
remains due			

Of *Troy* Weight.

EXAMPLES.

	1ſt,			2d.			3d.		
	oz	*dw*	*gr*	*oz*	*dw*	*gr*	*oz*	*dw*	*gr*
From	62	11	14	56	13	16	14	15	11
Take	35	10	19	22	4	18	6	16	2
	27	0	19						

Avoirdupois

Avoirdupoiſe Weight.

EXAMPLES.

	1ſt.			2d.			3d.		
	cwt	*qrs*	*lb*	*lb*	*oz*	*dr*	*lb*	*oz*	*dr*
From	62	2	24	18	3	2	16	10	1
Take	4	3	16	11	12	1	10	9	2
Diff.	57	3	8						

Apothecaries Weight.

EXAMPLES.

	1ſt.				2d.			
	oz	*dr*	*ſc*	*gr*	*oz*	*dr*	*ſc*	*gr*
From	45	3	2	11	36	2	1	10
Take	8	6	2	12	9	6	0	11
Diff.	36	4	2	19				

	3d				4th				5th			
	oz	*dr*	*ſc*	*gr*	*oz*	*dr*	*ſc*	*gr*	*oz*	*dr*	*ſc*	*gr*
From	35	2	1	12	16	1	0	10	18	2	1	11
Take	10	1	0	9	6	2	1	11	12	1	0	12
Diff.	25	1	1	3								

Of

Of *Land* Meaſure.

EXAMPLES.

Examples	1ſt.			2d.			3d.			4th.		
	A	*r*	*p*	*A*	*r*	*p*	*A*	*r*	*p*	*A*	*r*	*p*
From	12	2	12	16	3	16	26	1	6	30	0	8
Take	6	3	16	11	3	22	18	1	26	16	1	16
Diff.	5	2	36									

Of *Wine* Meaſure.

EXAMPLES.

	1ſt.			2d.			3d.			4th.		
	T	*hd*	*g*	*T*	*hd*	*g*	*g*	*qts*	*p*	*g*	*qts*	*p*
From	3	3	12	6	3	12	12	2	1	65	2	1
Take	2	2	18	2	2	26	10	0	1	10	3	0
Diff.	1	0	57									

Wincheſter Meaſure.

EXAMPLES.

	1ſt.			2d.			3d.			4th.		
	hds	*g*	*q*	*A.B.*	*f*	*g*	*B.B.*	*f*	*g*	*hds*	*g*	*qts*
From	16	11	12	16	1	3	45	1	7	40	3	2
Take	10	9	1	13	2	1	12	1	3	34	1	3
Diff.	6	2	1									

Of *Dry* Meaſure.

EXAMPLES.

	1ſt.			2d.			3d.			4th.		
	Ch	*bu*	*p*	*Ch*	*bu*	*p*	*qrs*	*bu*	*p*	*qrs*	*bu*	*p*
From	16	1	3	36	5	2	18	7	1	25	1	2
Take	11	2	1	29	3	0	11	1	2	16	1	3
Diff.	4	35	2									

Of *Time.*

EXAMPLES.

	1ſt				2d					3d				
	D	*h*	*m*	*ſec*	*W*	*d*	*h*	*m*	*ſec*	*w*	*d*	*h*	*m*	*ſec*
From	36	15	21	26	13	3	10	16	11	16	1	12	10	15
Take	25	16	32	30	16	2	18	46	21	13	0	14	5	16
Diff.	10	22	48	56										

Of *Motion.*

EXAMPLES.

	4th.			5th.			6th.			7th.		
	°	′	″	°	′	″	°	′	″	°	′	″
From	36	11	14	45	3	12	60	16	8	51	12	6
Take	25	10	16	13	20	43	45	18	26	36	14	21
Diff.	11	0	58									

Queſtions

Queſtions to exerciſe Addition and Subtraction.

1ſt A man was born in the year 1705; I demand his age in the year 1753?

2d A Caſtle was begun in the year 1450, and was 56 years in building; I demand in what year it was finiſhed, and how many years it is ſince?

3d James borrowed of Peter 965 pounds, of which he paid him at one time, 260 pounds, at another time 320 pounds, 10 ſhillings, and at another time 125 pounds, 12 ſhillings, and ſix pence; I demand how much remains unpaid?

4th What ſum is that, which taken from 653 pounds, leaves 425*l* 12*s* 8*d* ¾?

5th There were 5 bags of Money, containing as follows; viz. the firſt bag 125*l*, the ſecond bag 135*l* 7*s* 6*d* the third bag 200*l* 10*s*, the fourth bag 235*l* 12*s* 10*d*, and the fifth bag 300*l*; which were to be paid to ſeveral perſons, but one of them being loſt, there were only 780*l* 17*s* 6*d* paid; I demand which bag was wanting?

Of MULTIPLICATION.

MULTIPLICATION is a ſhort way of performing many Additions of the ſame numbers, as may be ſeen by comparing ſome examples in this rule, with others in Addition; to which I ſhall occaſionally refer.

Multiplication conſiſts of three parts.

1ſt The *Multiplicand*, or number to be Multiplied.

2d The *Multiplier*, or number by which you Multiply.

3d The *Product* or Total; which is juſt ſo many times the Multiplicand, as there are units in the Multiplier.

But before you can proceed any further, the following Table muſt be got by heart.

The TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

BY the above table, we readily find the product of any two numbers (neither of them being above twelve) multiplied by the other; and is found in this manner, viz.

FIRST, look for any two numbers propoſed, in the 1ſt column towards the left hand; that done, next find the other number in the uppermoſt or top column; then leading your eye, from the figure in the left hand column, directly acroſs the table, till you come exactly under the other figure in the upper column, you there find the product: as for example.

THE product of nine, multiplied by ſix is required; to find which, firſt look for 6 in your left hand column, and 9 in the upper one; which having found, guide your eye from 6, directly acroſs the table, till you come under 9, and there you find 54 which is the number ſought.

N. B. Had you taken your 9, in the left hand column, and your 6, in the upper one, it would have given you the ſame product.

SECTION 2d.

WHEN two numbers are given to be multiplied, one by the other, (if they don't conſiſt of an equal number of figures) it is moſt convenient to place the leaſt number lowermoſt, which is called your *Multiplier*; and the upper one of courſe, will be your *Multiplicand*. Having firſt carefully placed them, units under units, tens under tens, &c. then proceed to multiply them, as your table directs.

EXAMPLES.

	1st.		2d	3d
	pounds		*guineas*	*crowns*
Multiply	1 3 0 7	Multiplicand	4 2 6 3	5 7 9 2 6
By	2		4	5
	2 6 1 4	product		

4th	5th	6th	7th
7 1 5 4 8	3 8 2 4 6	9 7 6 3 2	4 2 7 6 5
7	8	1 1	1 2
			5 1 3 1 8 0

There are 6 purſes, each containing 2358 pounds, I demand how many pounds in the whole.

N. B. This proves the 7th example, in Addition of whole numbers.

SECTION 3d.

WHen your Multiplier conſiſts of more figures than one; you muſt multiply your Multiplicand, by each figure in the Multiplier ſeparately, placing the firſt figure of each product, exactly under its Multiplier; which ſeveral products being added together gives the total product.

N. B. When theſe figures 1 and 0, or 1 and 1, or 1 and 2, happen to ſtand next one another, in the Multiplier, you may multiply by both at once; as in the 7th example of the laſt ſection. See example 7th in this

EXAM-

EXAMPLES.

1ſt	2d	3d	4th
32457	436792	743278	210478
17	48	36	49
227199	3494336		
32457	1747168		
551769	20966016		

5th	6th	7th
247890	476273	3270486
423	322	3104
	952546	13081944
	952546	32704860
	1428819	9811458
	153359906	10151588544

8th	9th	10th
3756421	2435683	23471687
4116	5127	70812

S. THE reaſon why you carry at tens here, I perfectly underſtand; but the reaſon why the firſt figure of each product, ſhould be placed under its multiplier, I muſt beg of you to explain to me.

M. The very ſame ſteps by which you traced the one, would (if you had purſued a little further) have led you to the other, which is this; in the ſixth example of this ſection, the firſt figure in the Multiplier is but ſimply 2; therefore two times three units, is but ſix units, which I ſet down exactly under my Multiplier, viz. two, and ſo proceed till that line is finiſhed; I next begin to multiply by the 2d figure in my Multiplier, which is likewiſe two, but it is two tens; ſo that notwithſtanding as before (I ſay) two times three is ſix; yet by ſetting it under its Multiplier, which is the place of tens, it becomes ſixty, as it ought; twenty times three units, or three times twenty; or two tens, being likewiſe ſixty; having finiſhed this line, I next proceed to multiply by my laſt figure viz. three, which from the place it ſtands in, is three hundred; ſo that if I had placed my nine, which is the product of three times three, below my ſix, in the firſt line, it would have been but nine units; or if below my 6 in the 2d line, 9 tens; whereas it is 900; three hundred times three units, or three times three hundred being 900; and ſo of all the reſt.

SECTION 4th.

WHEN your Multiplicand, or Multiplier, do either, or both of them, end with a cypher or cyphers:

So many cyphers as you have in your Multiplier, ſet down directly under them, in the firſt product, and then begin to multiply by your firſt ſignificant figure.

gure in your Multiplier (which in the 1ſt example in this ſection is 2, ſaying two times nought is nought, &c. and ſo proceed till you have finiſhed that line. Next begin to multiply by your laſt figure, which is 4, placing the firſt figure of the product exactly under it; and having finiſhed that line, add the two products together, which will give you the total product.

Or neglecting ſuch cyphers in both; multiply by the ſignificant figures only; and when your multiplication is finiſhed, annex to the right hand of ſuch product, the cypher or cyphers, with which one, or both of your firſt numbers given, did end; which will alſo be the total product as before.

EXAMPLES.

	1ſt		2d
1ſt way	746300	2d way	746300
	4200		4200
	149260000		14926
	2985200		29852
	3134460000		3134460000

3d	4th
654000	536000
5208	3200

SECTION 5th.

WHEN cyphers are placed between the ſignificant figures in the Multiplier, they muſt be omitted in the operation, regard being had to the firſt figure of every particular product, as before.

EXAMPLES.

1ſt	2d	3d
346427	47489076	843269259
6004	700064	705003
1385708		
2078562		
2079947708		

SECTION 6th.

WHEN the multiplier is ſuch a number, that you can foreſee any two figures which being multiplied together will produce the ſaid number, it is ſhorter to Multiply the given number, firſt by one of thoſe figures, and that product again by the other; as you ſee by the following examples.

EXAMPLES.

1ſt.	2d.
Multiply 436792 by 48	Multiply 7459764 by 72
8	9
3494336	67137876
6	8
20966016	537103008

N. B. By this method we not only ſave a line, but the trouble of adding the products together, as may be ſeen by comparing the 1ſt example in this ſection, with the 2d example in ſection 3d.

Multiply 574027 by 54 | Multiply 740769 by 84

IN the 1ſt example of this ſection, the Multiplier being 48; I firſt multiply by 8, which gives a product equal to 8 times the Multiplicand; and becauſe 6 times 8, is 48: I again multiply the ſaid product by 6, which 2d product you will find to be equal to the Multiplicand, multiplied by 48.

Note 2d. This method of multiplying is of great uſe in practical, or compound multiplication: and may be extended much beyond the compaſs of your Multiplication table.

Of Compound Multiplication.

THIS kind of Multiplication (as I obſerved before, is very uſeful in ſolving many queſtions, as will appear by the examples following; to perform which, no other directions are neceſſary, regard being had to thoſe already given in addition of divers denominations; viz. how many of one denomination, are equal to one of another.

EXAMPLES

EXAMPLES.

A grocer bought 8 hogſheads of ſugar, each weighing 3 *cwt.* 2 *qrs.* 7 *lb.* I demand the weight of the whole.

	cwt.	*qrs*	*lb*
	3	2	7
			8
Anſ.	28	2	0

A Nobleman allows for the uſe of his houſe, each week, 3 *chal.* 12 *buſh.* 3 *pecks* of coals: what quantity does he conſume in the year?

A gentleman propoſes taking a journey, which at 45 miles, 4 furlongs, and 10 poles, each day will take him 27 days to perform it in, I demand the number of miles he travelled at that rate.

m	*f*	*p*	
45	4	10	
		9	
409	6	10	in 9 days
		3	
1229	2	30	the anſwer.

A ſilver-ſmith bought 9 old ſilver tankards, each weighing 1 *lb.* 10 *oz.* 9 *dw.* 8 *gr.* what did the whole weigh?

SECTION 1ſt.

WHEN the number of Integers, whoſe price are required, exceed the product of 12, multiplied by 12, but leſs than 157:

Rule. Firſt multiply the price of one Integer by 12, which produces the price of twelve Integers; and then multiply that product again by 12, will give you the price of 144; and next multiplying the price of one Integer, by the remaining number of Integers, ſet down the 1ſt product under your laſt line: which being added together, gives you the price of the whole, as you will ſee by the 1ſt example of this ſection.

2dly. When the number of Integers exceed 156, or if more hundreds than one;—

FIRST multiply the price of one Integer by 10, which gives the price of ten; again multiply that product by 10, which will give the price of 100; which laſt product being multiplied by the number of hundreds, viz. 1, 2, 3, 4 or 5, &c. according to the number of hundreds wanted; you will have the price of all the hundreds. Then for the remaining odd number (if any) you may take part of any of the former products; or multiply the given price, by the number wanted; which being added together, gives you the price of the whole, as you will find more clearly explained by the operation of the 2d example in this ſection.

E X A M P L E S.

1ſt A Merchant bought 156 caſks of brandy at £2 15s 8d per caſk, what muſt he pay for the whole?

£	s	d	
2	15	8	the price of one caſk.
	12		
33	8	0	price of twelve do
	12		
400	16	0	price of one hundred and forty four.
33	8	0	price of twelve do
£434	4	0	price of the whole.

2d. What is the value of 425 pieces of cloth at 2*l* 7*s* 6*d* per piece?

	£	*s*	*d*	
	3	7	6	the price of one piece
			10	
	33	15	0	price of ten pieces
			10	
	337	10	0	price of one hundred
			4	
	1350	0	0	price of - 400
Twice the price of 10 is	67	10	0	the price of 20
5 times the price of 1 is	16	17	6	the price of 5
	1434	7	6	price of - 425 pieces.

IN the above queſtion, I firſt ſet down the price of one piece, and then find the price of the four hundred in this manner, viz, firſt I multiply the price of one Integer, by ten, which gives the price of ten; and multiplying that product again by ten, gives the price of 100, which multiplying again by 4, gives the price of 400. Then for the price of the odd 25 pieces, I multiply the price of ten by 2, which gives the price of 20, and laſtly multiplying the price of 1, by 5, I have the price of 5; which three laſt products being added together give the price of the whole.

3d. If 2435 Men were to be cloathed, and 2*l* 3*s* 4*d* allow'd for the cloathing of each man; what would the whole coſt?

Scho. THE method of applying Multiplication in the foregoing examples, has given me ſome notion of its uſe; but pray is there no other way of ſolving thoſe queſtions?

Maſt. Yes; both by Practice and the Rule of Three.

S. Which of them is ſhorteſt?

M. Sometimes Practice, and ſometimes Multiplication; and therefore chiefly depends on the ſkill of the artiſt, when to apply the one, and when the other; but for the beſt general rule, obſerve the following.

General Rule when to apply Practice, *and when* Multiplication.

WHEN any of the numbers conſiſt entirely of Integers, and not exceeding 156: the ſhorteſt way is then by Multiplication; But when the Integers exceed that number, particularly if a very large one (as in the 2d and 3d examples aforegoing) then it is more conveniently perform'd by Practice; as will appear by comparing the operation of the ſame queſtions in each of thoſe rules (when you have learnt them) together.

But leaſt the learner, (for want of time, or from any other cauſe) ſhould be obliged to ſtop ſhort; I thought it neceſſary to teach him how to make the moſt of what he had learnt; by ſhewing him the uſe and application of each rule, in its utmoſt extent.

Of DIVISION.

BY Diviſion we diſcover, how often one number is contained in another, and what remains; and is a ſhort way of performing ſeveral Subtractions.

IN Diviſion there are three numbers; two of which are given to find a third, viz.

The two given numbers are { The Dividend, or number to be divided.
The Diviſor, or number by which you divide.

The Quotient, or number ariſing from ſuch diviſion, and which ſhews how often the Diviſor is contained in the Dividend.

Note, After the diviſion is finiſhed, there happens often to be a remainder, which muſt be always leſs than the Diviſor, and of the ſame name or kind, with the Dividend.

THERE are alſo two ſorts of ſimple Diviſion, viz. Short Diviſion, and Long Diviſion.

Of SHORT DIVISION.

SHORT Diviſion is commonly made uſe of when the Diviſor doth not exceed 12, as in the following.

EXAMPLES.

Divide 387 by 4

Dividend
Diviſor 4) 387 (

Quot. 96 : 3 remdr.

2d Divide 4236 by 5

Dividend Quot.
Diviſor 5) 4326 (865

1 remdr.

3d Divide 30741 by 6
4th. Divide 79045 by 7
5th. Divide 497604 by 7
6th. Divide 780432 by 8
7th. Divide 9470653 by 9
8th. Divide 2364037 by 10
9th. Divide 1974865 by 11
10th. Divide 50479804 by 12

I have wrought the 1ſt and 2d Examples, to ſhew you how the operation is perform'd; which is in this manner. Firſt ſetting down your Dividend, and placing your Diviſor to the left hand of it as you ſee; I aſk how often 4 is contained in 38, which I find to be 9 times, and 2 over; I therefore ſet down my 9 under my 8, and carrying the 2 in my mind, to the 7, which is the next figure in my dividend; I again ſeek how often 4 is contained in 27, which I find to be 6 times, and 3 over; I therefore ſet down my 6 to the right hand of my 9: and becauſe there are no more figures in my Dividend to which I might carry my 3, which remained; I ſet it down at ſome diſtance to the right of my quot. making a mark of diſtinction between them, and the whole is finiſhed.

Note, The 2d example is performed in the ſame manner, although I have placed the quot. and remainder in a different order; This firſt way however is the moſt common, in Short Diviſion.

WHEN the ſignificant figures in the Diviſor do not exceed twelve; with any number of cyphers whatever annext to it, you may then cut off ſuch cypher, or cyphers, from your Diviſor, and alſo an equal number of figures from the right hand of your Dividend; and then divide by your ſignificant figures only; always remembring (when your Diviſion is finiſhed) to add the figures which you cut off in your Dividend, to the the remainder at laſt; if nothing remain, then thoſe figures cut off in your Dividend, are the whole remainder.

EXAMPLES.

1ſt Divide 34260 by 20
2|0) 3426|0 remainder
1713 quot.

2d Divide 21073 by 40
4|0) 2170 (3
quot. 542 23 remainder.

3d Divide 504731 by 80

4th Divide 734267 by 900
5th Divide 970458 by 1000
6th Divide 237560 by 120
7th Divide 5432750 by 1200
12|00) 54327 (50
quot. 4527 350 remdr.

Divide 896584321 by 11000

I ſhall here ſhew you how the 7th example is perform'd, by which you will underſtand how to work any of the reſt. Firſt then having cut off my two cyphers in the Diviſor, and alſo an equal number of figures from my Dividend: I then proceed to divide the remaining part of my Dividend, by 12: and having finiſhed the Diviſion, I have 3 for a remainder, to which bringing down my 50, which I cut off in my Dividend, and placing it to the right hand of my 3, which remained in diviſion; gives the whole remainder 350, as appears by the work itſelf.

Of LONG DIVISION.

SECTION 1ſt.

WHEN the ſignificant figures in the Diviſor exceed 12, it is then called Long Diviſion; being obliged for an help to the memory, to multiply the Diviſor by one ſingle figure at a time, till you have multiplied the whole Diviſor: Then ſubtracting that product, from that part of the Dividend you ſought it out of, in order to find the remainder: you next to the right hand of the ſaid remainder, bring down the next figure in your Dividend; which operation muſt be continued to every Quotient figure; as you will ſee by the following.

EXAMPLES.

1ſt Divide 5840 by 48

Dividend Quotient
Diviſor 48) 5840 (121
48
104
96
80
48
32 Remainder

IN performing the foregoing operation, I firſt aſk how often the Diviſor 48, is contained in 58, which are the firſt two figures in the Dividend: and finding it to be only once, I therefore ſet down one in the Quotient, and alſo ſet the 48, under the 58, in the Dividend; and ſubtracting the one from the other I find a remainder of ten; then bringing down the next figure of the Dividend, which is 4, I again ſeek how often the Diviſor is contained in 104, which I find twice: and ſetting my 2 to the right hand of my 1 in the Quotient, I multiply my Diviſor again by 2; and place the product regularly under the 104: which being ſubtracted from it, leaves a remainder of 8; and laſtly, bringing down my nought, I enquire again, how often 48 in 80; which being but once, I ſet my 1 in the Quotient, and ſubſcribe my Diviſor below 80; which having ſubtracted from it, the laſt remainder is 32; ſo that I find 48 to be contained in 5840, 121 times, and 32 over.

Note, If after you have brought down a figure from your Dividend to your remainder, the number ſhould ſtill be leſs than your Diviſor; you muſt ſet a nought in your Quotient, and bring down the next figure in your Dividend: if the number be ſtill leſs than your Diviſor, you muſt ſet another nought in your Quotient, and ſo proceed till you have a Dividend large enough to contain your Diviſor.

HAVING now ſhewed you how the operation is perform'd; I ſhall next teach you the application of it to Addition, inſtead of pointing.

2d Dividend 3d *lb.*
Diviſor 16) 76 (4 quot. { 28) 168 (6
64 168
12 ...

4th *grs* 5th *cwt*
4) 18 (4 { 20) 97 (4
16 80
2 17

THE four examples above, repreſent the reſpective ſum, which each of the four lines amount to in the

3d example in Addition of Avoirdupoiſe weight. viz. the 2d example, the line of ounces; the 3d, that of pound weights; the 4th, that of quarters; and the 5th, the line of hundreds.

HAVING added up the 1ſt line in the 1ſt example in Addition, viz. ounces, as if they were Integers; I find it amounts to 76, which I place for a Dividend, as you ſee in the 2d example aforegoing; and becauſe 16 ounces are a pound weight; I therefore place 16, as my Diviſor; which having divided by, I find to be contained in 76, four times, and a remainder of 12; which being 12 ounces, I therefore ſet it down below the line of ounces, and the quotient which is 4 pound, I carry to the line of pounds; and having added it up, find the ſum to be 168: to know then how many quarters are contained in 168 pounds, I divide them by 28, as you ſee in the 3d example above; and find it to be 6 times, and nothing over; I therefore ſet down nought, under the line of pounds, and carry my 6 quarters to my line of quarters, and ſo proceed: always ſetting down your remainder below, and carrying your Quotient to the next line, till you come to the laſt denomination towards your left hand, at which (as has been obſerved before) you muſt carry at tens.

SECTION 2d.

WHEN the Diviſor is ſuch a number that any two figures, being multiplied together, will produce the ſaid Diviſor; it is ſhorter firſt to divide the Dividend by one of thoſe figures: and again, that Quotient by the other: as you ſee by comparing the 1ſt of the following examples, with the 1ſt example in long Diviſion, where inſtead of dividing by the Diviſor 48 all at once; I firſt divide by 8, and that Quotient again by 6; ſix times 8 being 48.

EXAMPLES.

1ſt. Divide 5840 by 48
8)5840(
6)730
Quotient 121 - 32 remainder

2d. Divide 24836 by 64
8)24836(
8)3104 : 4 rem.
388 Quot.

3d. Divide 32745 by 54
9)32745(
6)3638 - 3
606 - 21

Scho. This is indeed, not only much ſhorter; but much eaſier than dividing the long way: but I don't underſtand, how you come by the ſeveral remainders, in the foregoing exmaples.

Maſt. After you have divided by the firſt figure if there be a remainder, it is juſt ſo many units, as the figure expreſſes; and the Quotient ſo many of that figure you divide by; ſo in the ſecond example, where you divide firſt by 8; your firſt Quotient is ſo many times 8; and your remainder 4 units: conſequently in the 1ſt example, (where your 1ſt Diviſor was likewiſe 8) your firſt Quotient will be ſo many times 8 alſo, which being again divided by 6, your laſt Quotient will be ſo many times 48, as there are units in your Quotient; and your laſt remainder being 4, is 4 of your firſt Quotient; viz. 4 times 8, which is equal to 32; therefore ſet it down under, as you ſee.

LASTLY, in the third example; the firſt remainder being 3, is ſimply 3 units, which I ſet down; but the laſt remainder (which was 2) is two nines, which is 18: to which adding my firſt remainder 3, the whole remainder is 21, which operation you may prove by dividing the 2d and 3d examples the long way.

Here follow ſome more examples in Long Diviſion, for the exerciſe of the learner.

Diviſor 1200) 54327 50 (4527
4800
6327
6000
3275
2400
8750
8400
350 Remainder

Compare this operation with that of the 7th example in the 1ſt ſection in ſhort Diviſion.

7th 32700) 74320167 (
8th 46045) 87023650 (
9th 50768) 721400572 (
10th 859624) 307867329 (

Note, When your Diviſor conſiſts of any number of ſignificant figures whatever, with a cypher, or cyphers, annexed, you may ſhorten the operation according to the rule given in the firſt ſection of Short Diviſion.

S. This is indeed a very curious, and very useful contraction; as it not only answers every purpose that Long Division does; but the operation is much leſs complex, although the reaſon on which it is founded ſeems (to me) far more myſterious; and I therefore beg of you to explain it to me.

M. Firſt then you muſt note, that any two numbers being multiplied, by one and the ſame ſum, the products thence ariſing, will have the ſame proportion to each other, as the firſt given numbers had, before they were multiplied. As for inſtance, 6 and 12 being each multiplied by 3, the products will be 18, and 36: hence it appears, that as 12 was equal to 2*ce* 6: ſo likewiſe is 36 equal to 2*ce* 18. Conſequently then, any two numbers, which will divide equally, by one and the ſame ſum without a remainder; their reſpective Quotients, will bear the ſame reaſon, or proportion to each other, as thoſe numbers had which were given to be divided.

For a proof of this

LET us ſuppoſe the Dividend, and Diviſor in a queſtion in Diviſion, to be the two numbers given: viz. the Dividend 45, and the Diviſor 15: and let the number by which each of them is to be divided, be 3; the Quotients thence ariſing will be 5 and 15; I ſay then, that the Quotient of 15, divided by 3, ſhall be juſt as often contained, in the Quotient of 45, divided by the ſame figure; as the firſt Diviſor 15, was contained in the firſt Dividend 45; for as 5 is contained in 15, three times; ſo likewiſe is 15 three times contained in 45.

THESE things being premiſed, the reaſon for cutting off as many figures from the right hand of your Dividend, as you do cyphers from your Diviſor, will appear clear from the following conſiderations.

AND firſt, that as every figure increaſes in a ten-fold proportion towards the left hand; ſo of courſe, it muſt decreaſe in the ſame proportion towards the right. To make this ſtill more plain: let us again take a view of the 1ſt example in the 1ſt ſection of ſhort Diviſion. There you ſee, that by cutting off the cypher from the right hand of the Diviſor, and alſo one from the right hand of the Dividend, you therefore ſhift every figure one place further towards the right hand, and conſequently make it ten times leſs than it was before. Each figure then, being ten times leſs, it neceſſarily follows, that the whole muſt be ten times leſs alſo; and therefore according to the foregoing obſervation, muſt have the ſame proportion to each other as before; ſo that the remaining figures in the Dividend, being divided by the remaining figure or figures in the Diviſor, will give the ſame quotient, as if the whole Dividend, had been divided by the whole Diviſor; as has been ſhewed by the 7th example in the 1 ſection of Short Diviſion, and the 6th example in Long Diviſion.

FOR the very ſame reaſon then, that cutting off one cypher in your Diviſor, and one figure in your Dividend is equal to dividing by 10; ſo cutting off two cyphers; is equal to dividing by 100, and three to dividing by 1060, &c.

S. This definition, ſeems very clear, when the Diviſor, and Dividend end each with a cypher or cyphers, but will the ſame reaſon hold good, when any, or all the figures cut off in the Dividend, happen to be ſignificant ones?

M. Moſt certainly, the only difference being this: that where the number of cyphers cut off in the Dividend, are equal to thoſe cut off in the Diviſor: then the Dividend and Diviſor are equally divided by 10, 100, 1000, &c. (according to the number of cyphers cut off in each) without a remainder: but when any or all the figures ſo cut off in the Dividend happen to be ſignificant ones; then ſuch figures, are the remainder of the Dividend, after being divided by 10, 100, 1000, &c, For there are many queſtions in Diviſion, where the Diviſor and Dividend are incapable of being divided equally by 10, or any other number whatſoever, without a remainder; as I ſhall more fully prove when I come to treat of Vulgar Fractions.

S. I preſume then, that in all queſtions, where the Dividend and Diviſor, cannot be equally divided by one and the ſame number, without a remainder, that the Quotients ariſing from ſuch Diviſion, cannot have the ſame proportion to each other, as the Diviſor and Dividend firſt given.

M. True, the whole numbers ariſing in the Quotient of the Dividend, cannot bear the ſame proportion to the Quotient of the Diviſor, as the Diviſor and Dividend firſt given had to each other: but if to the whole numbers in the quotient of the Dividend, you annex the remainder, then will it bear the ſame proportion to the Quotient of the Diviſor, as the Diviſor and Dividend firſt given had to each other.

THE proof of this I muſt alſo be obliged to defer till I have brought you into Fractions, from whence, I ſhall refer you to ſome queſtions in this rule.

S. How is Diviſion proved?

M. Multiplication and Diviſion, mutually prove each other; for if you divide the product of any queſtion in Multiplication, by the Multiplier; the Quotient will be equal to the Multiplicand: and again, if you multiply the Quotient of any queſtion in Diviſion, by the Diviſor: the product will be equal to the Dividend.

Here follow ſome queſtions to exerciſe *Addition, Subtraction, Multiplication* and *Diviſion;* which if the young Arithmetician thinks too difficult; he may (if he pleaſes) paſs over: till his reaſon be a little riper, and his conceptions more enlarged: as they are no link of that chain, which connects the foregoing and following rules together.

Queſtions to exerciſe *Addition, Subtraction, Multiplication,* and *Diviſion.*

1ſt What number is that which being multiplied by 3, and divided by 12; will give you 6 for the Quotient?

2d What number divided by 12, and the Quotient multiplied by 3, will give 6 for the product?

3d What number multiplied by three,
To eighty four will equal be?

4th What is that number, which divide
By twice fifteen, and ten beſide;
Then to the Quotient if there be
Nine added, makes it fifty three?

5th Pray can you on a number fix,
Which multiply'd by ſeven times ſix;
And then divide by thrice eleven,
Shall give the Quotient twelve times ſeven?

6th Pray what's that number, tell me friend;
By which divide this 23100 Dividend:
It's Quotient next divide by ſeven,
The laſt ſhall be twelve times eleven.

7th What number's that, if added be
The product to't, of nine times three;
From which ſubtract, five times eleven,
And next divide by twelve times ſeven;
The Quotient may'nt be leſs or more,
But equal juſt to twice three ſcore.

S. The ſeven foregoing queſtions are very curious and entertaining; but I cannot conceive any caſe, which poſſibly can happen in buſineſs; in which they can be of any real uſe.

M. They are not only of general, but of particular uſe.

1ſt Of general uſe, as they teach you how to reaſon methodically upon hard problems; by which means you are able to form clear, and diſtinct notions of things; to diſtinguiſh the difference between queſtions which are really difficult, and thoſe which only appear ſo, by the critical manner in which they are often propoſed; and conſequently to reduce the latter to more ſimple propoſitions.

2d They are of particular uſe; as there are many caſes, in which they may become not only rational (but very natural) queſtions; both of which, I ſhall prove, by the laſt queſtion aforegoing; which being reduced into proſe, is no more than ſimply this, viz.

WHAT number is that, to which if you add 27, and from that ſum ſubtract 55, rhe remainder being divided dy 84, will give 120 for the Quotient?

TO ſolve this, I firſt multiply the Quotient 120, by the Diviſor 84, which (according to the rule given for proving Diviſion) produces the Dividend 10080: which Dividend, was compoſed of a certain unknown number, to which 27 were added, and from that ſum 55 ſubtracted. As therefore, the ſum ſubtracted, was greater than that which was added by 28, I add the ſaid 28 to 10080 which gives me the 1ſt unknown number, viz. 10108, which was the anſwer required. See the work.

84
120
———
10080
28
———
10108 number reqrd.

proof 10108
27 added
———
10135
55 ſubtracted
———
divſr. 84) 10080 (120 quot.
84
———
168
168
———
0

HAVING, I hope by the fore-going operation, fully proved the general uſe of ſuch queſtions, namely in reducing thoſe to more ſimple propoſitions, which (from the critical manner in which they were propoſed) might ſeem difficult: I ſhall next proceed to give you one inſtance, out of many, in which they may be of particular uſe; and in what caſes they become not only rational, but very natural queſtions, as in the following caſe.

A certain gentleman fitted out a privateer, which he commanded in perſon, and figuring to himſelf (like other adventurers of that kind) what an immenſe fortune he would make; determined to run all chances himſelf: and therefore agreed to give each of his crew, which in all conſiſted of 84, 120£ for the cruize. He took ſeveral ſmall prizes, which when ſold amounted to a certain ſum of money. The Broker's commiſſion for ſelling them, at a certain rate per cent, came to 55 pounds, which being deducted from the whole amount, the Captain found the remainder would not pay off his men by 27 pounds, which ſum he was obliged to pay out of his own pocket. I demand the whole amount which the prizes ſold for?

BY this laſt method of propoſing this queſtion, you ſee it is made more ſimple ſtill: for by making the ſeveral numbers given in it, to repreſent certain things; we thereby fix a meaning on the whole; which reduces it, from a ſeeming difficult, to a very eaſy queſtion.

THE reader may perhaps expect (like moſt other authors) that I ſhould here have given them ſome practical queſtions, ſhewing him how to divide any ſum of money, amongſt any certain number of men, giving each an equal ſhare. I muſt therefore tell him, that ſuch examples would be very improperly claſſed in this place, ſince the learner muſt be entirely at a loſs how to manage the remainders when there happens to be any, till he has firſt attained ſome knowledge in Reduction, under which head he will find them.

CHAP. II.

Of REDUCTION.

REDUCTION teacheth, to bring or reduce numbers of one denomination, to other numbers of another denomination, retaining the ſame value: and is either aſcending, or deſcending.

WHEN greater denominations are to be brought into leſſer ones: as pounds into ſhillings, tons into hundred weights, &c. This is called Reduction deſcending, and is performed by Multiplication.

WHEN ſmall names are to be brought into great; as ſhillings into pounds, hundred weights into tons, &c. then it is called (though improperly) Reduction aſcending: and is performed by Diviſion.

Note, Whether you multiply or divide, it muſt always be by as many of the leſſer, as make one of the greater.

BEFORE you proceed further, I thought it neceſſary to give you the following table of *Alliquot parts*, to be got by heart: which (although few or no authors give before the rule of *Practice*) I thought equally uſeful in this place, and may therefore ſerve for both.

The TABLES.

Shewing what Pence are (=) equal parts of a Shilling; and what Shillings and Pence are (=) equal parts of a Pound.

1ſt.

d.		1*s.*
6	=	$\frac{1}{2}$
4	=	$\frac{1}{3}$
3	=	$\frac{1}{4}$
2	=	$\frac{1}{6}$
$1\frac{1}{2}$	=	$\frac{1}{8}$

2d.

s.	*d.*		1*l.*
0	0		$\frac{1}{2}$
6	8		$\frac{1}{3}$
5	0	=	$\frac{1}{4}$
4	0	=	$\frac{1}{5}$
	4		$\frac{1}{6}$
2	6	=	$\frac{1}{8}$
2	0	=	$\frac{1}{10}$
	8	=	$\frac{1}{12}$

3d.

s.	*d.*		1*l.*
1	4	=	$\frac{1}{15}$
1	3		$\frac{1}{16}$
1	0		$\frac{1}{20}$
0	8	=	$\frac{1}{30}$
0	6		$\frac{1}{40}$
0	4	=	$\frac{1}{60}$
0	3	=	$\frac{1}{80}$
0	2		$\frac{1}{120}$

REDUCTION *Deſcending.*

Of MONEY.

Example 1ſt. In 54£ how many ſhillings and pence?

£
54
20

1080 ſhillings
12

12960 pence

Queſtions	*Anſwers*
2d In 5 £ how many ſhillings, pence, and farthings.	100s. 1200*d*. 4800 *qrs*
3d In 7£ 14s 9d¼ how many ſhillings, pence, and farthings.	154s. 1857*d*. 7429 *qrs*
4th In 15 crowns how many ſhillings and pence?	75 *s*. 900*d*.
5th In 25 £ how many crowns and ſhillings?	100 crowns. 500*s*.
6th Reduce 150 crowns to ſhillings and groats.	750 crowns. 2250 *grs*.
7th How many ſhillings and pence, are there in 21 guineas?	441*s*. 5292 *pence*.
8th How many crowns and ſixpences, are in 35 pounds.	140 crowns. 1400 ſixpences.

LEAST by the operation of the 1ſt example, you ſhould not yet thoroughly underſtand how the reſt are to be perform'd; eſpecially where there are ſhillings given with the pounds, or ſhillings, pence, and farthings; I ſhall here ſhew how the 3d example is wrought.

£ *s* *d*
7 14 9¼
20

154 ſhillings
12

1857 pence
4

7429 farthings

IN performing this operation, I firſt ſet down the ſum given to be reduced, as you ſee; and becauſe 20 ſhillings make one pound, I therefore multiply the 7 pounds by 20, and take in the 14 ſhillings in this manner, viz. The 4 being four units, I take in with my thought, which ſtands likewiſe in the place of units: and the 1 which is one ten, I take in when I multiply by 2, which gives the whole number of ſhillings. I next multiply the number of ſhillings by 12, taking in my 9 pence, which gives the number of pence: and laſt of all, I multiply the pence by 4 (four farthings being 1 penny) taking in my 1 farthing, as you ſee by the work.

Queſtions.

REDUCTION *Aſcending.*

Queſtions.	*Anſwers.*
1ſt In 12960 pence, how many ſhillings and pounds.	542 as appears by the work.

12) 12960 (

2|0) 108|0 ſhillings.

54 pounds

2d In 4800 qrs. how many pence, ſhillings, and pounds.	1200*d.* 100*s.* 5*£.*
3d In 7429 qrs. how many pounds?	7*£.* 14*s* 9*d* ¼
4th In 900 pence how many ſhillings and crowns?	75*s.* 15 *crowns.*
5th In 500 ſhillings, how many crowns and pounds?	100 *crowns* 25*£*
6th In 2250 groats, how many ſhillings and crowns?	750*s.* 150 *crowns,*
7th 1323 pence, how many guineas and ſhillings?	21 *guineas.* 441*s.*
8th In 1400 ſixpences, how many pounds?	35 *£*

Note, The above examples and thoſe in Reduction Deſeending, prove each other.

REDUCTION

REDUCTION *Aſcending* and *Deſcending*.

IN 325 Shillings how many crowns and half crowns?

5] 325

65 crowns

2

130

TO ſolve this queſtion I firſt divide the number of ſhillings by 5, which gives the crowns, which being multiplied by 2 gives the half crowns.

2d In 100 crowns, how many ſhillings, and pounds?
Anſwer 500*s.* 25*l.*

3d In 60 guineas, how many ſhillings, crowns, and pounds? *Anſwer* 1260*s.* 252*cr.* 12*l.* 12.

4th In 6320 two pences, how many pounds?
Anſwer 52*l.* 13*s.* 4*d.*

5th In 1245 three pences, how many pounds?
Anſwer 15*l.* 11*s.* 3*d.*

6th In 1456 fourpences, how many pounds?
Anſwer 24*l.* 5*s.* 4*d.*

7th In 6312 ſixpences how many pounds?
Anſwer 157*l.* 16*s.*

8th In 2160 eight-pences how many pounds?
Anſwer 72*l.*

9th In 6543 half crowns, how many pounds?
Anſwer 817*l.* 17*s.* 6*d.*

10th In 3261 three and four-pences how many pounds?
Anſwer 543*l.* 10*s.*

11th In 3156 nobles, how many pounds?
Anſwer 1052*l.*

12th In 56*l.* 12*s.* 9*d.* how many three-pences? *Anſwer* 4531.

13th In 810 half crowns, how many crowns and pence? *Anſwer* 405 *crowns.* 12150*d.*

14th Reduce 84 crowns, into ſhillings and guineas. *Anſwer* 420*s.* 20 *guineas.*

15th Reduce 60 moidores into pounds. *Anſ.* 81*l.*

16th How many crowns, half crowns, and ſhillings, are in 260 pounds, and the number of each equal? *Anſwer* 611 of each and 6*s.* 6*d.* over.

17th Divide 427 pounds amongſt 4 men, and give each an equal ſhare. *Anſwer* 106*l.* 15*s.*

S. Pray how is the 16th queſtion aforegoing to be wrought?

M. Firſt, add your crown, half crown, and ſhilling together; which being reduced to the loweſt denomination mentioned, will be 17 ſix-pences: Then reduce your 260 pounds alſo into ſix-pences, and dividing the one by the other, you have the anſwer.

THE reaſon of this operation, I ſhall demonſtrate to you, by a ſimilar queſtion in Reduction of liquid meaſure, when we have got thither.

I ſhall here ſhew you how the 11th example aforegoing is wrought the common way, and likewiſe how it is perform'd by aliquot parts; by which means you will ſee, how greatly the work may be ſhortened when the value of one of the Integers given to be reduced, happens to be any of thoſe equal parts of a pound which you find expreſſed in your table.

The common way, 3156 ſix and eight pences.
80 pence in each.

12)252480

2|0)2104(0

1052 pounds

By alliquot parts 6s. 8d $\frac{1}{3}$) 3156

1052 pounds

By the common way, I firſt ſee how many pence are in one noble, or 6s 8d; which (according to the method of working queſtions in Reduction deſcending) I find to be 80; I therefore multiply the number of nobles viz. 3156 by 80, which gives the pence in the whole. I next divide thoſe pence by 12, being the number of pence in one ſhilling, and the quotient is *ſhillings*: and laſtly dividing that quotient by 20, twenty ſhillings being one pound, the quotient gives the anſwer in *pounds*.

Note 1ſt. If after dividing by 12 there had been a remainder, ſuch remainder would have been *pence*: and if any, after dividing by 20; would have been *ſhillings*.

IN working the ſame queſtion by alliquot parts: I find by the tables, that 6s 8d, or one noble, is $\frac{1}{3}$ of a pound; wherefore dividing the number of nobles, viz. 3156 by 3, I have the anſwer in pounds at once. If there had been a remainder, it would have been ſo many times ſix and eight pence, viz. if one, then 6s 8d; if 2, thirteen and four pence.

Note 2d. IN all queſtions where the value of one of the Integers given to be reduced, happens to be an alliquot part of a pound, as is the 4th, 5th, 6th, 7th, 8th, 9th, 10th, and 11th examples aforegoing; then it is beſt to divide by ſuch parts; which will give you the anſwer in pounds at once: but when one of the Integers given, is no alliquot part, you muſt then perform the operation the common way, as before directed.

Note 3d. When pounds, ſhillings and pence, are given to be reduced into two-pences, three pences, or any other number of pence, which are an equal part of a ſhilling; you may often perform the operation ſhorter than by the common method; as I ſhall prove to you by working the 12th example afore-going, both ways; where it is required to reduce 56*£*. 12*s*. 9*d*. into three-pences.

£ *s* *d*
Common way 56 12 9
20

1132
12

3) 13593 (

4531 anſwer

£ *s* *d*
The other method 56 . 12 9
20

1132
4

4531 anſwer

BY the common way, I firſt multiply my 56 pounds by 20, and take in my 12 ſhillings; and then again multiply by 12 and take in my 9, which brings it all into pence, which being divided by 3, gives me the anſwer.

But by the ſecond method of working it.

AFTER having multiplied by 20, and taken in my 12s. as before, I then inſtead of multiplying by 12; multiply only by 4: four three-pences being one ſhilling, to which adding my three-pences which are contained in my nine pence, I have the anſwer in 3 pences, without the help of Diviſion.

AS I ſhall have occaſion when I come to treat of Vulgar-Fractions, to refer you to the laſt of the foregoing queſtions viz. the 17th: I ſhall likewiſe ſhew you how it is performed, by which time you need but very few further inſtructions to work any queſtion in Reduction. In the ſaid 17th queſtion, it is required to divide 427 pounds among 4 men, and to give each an equal ſhare.

4) 427

```
4) 427 (106 ¾ each man
   4
   ——
    27
    24
    ——
     3 pounds remainder
    20
    ——
4)  60 (15 ſhillings
    4
    ——
    20
    20
    ——
    ..
```

HERE having firſt divided my 427 pounds by 4, I find that each man will have 106 pounds, and 3 pounds remain over, which remainder I place under my Quotient, and ſet my Diviſor under it, which ſignifies that each man will have 106*l.* and ¾ to his ſhare. But becauſe I would know what the ſaid ¾ of a pound is equal in value to; I therefore multiply my 3 pounds which remained by 20, which brings it into ſhillings; and then dividing again by the four men, I find gives them 15*s*, per man more. So each man's whole ſhare will be 106£ 15*s*.

Of *Troy* Weight.

1ſt In 11*lb.* 10*oz.* 11*dwts.* how many grains?

Anſwer 68424*grains*,

2d A goldſmith having 66 Ingots of ſilver, each weighing 27*oz.* was minded to make them into ſpoons of 2*oz.* cups of 4*oz.* ſalts of 5*oz.* and ſnuff boxes of

30*oz.* and to have an equal number of each: the queſtion is, what number of each he muſt have?

Anſwer 11, and 8 over.

3d In 21000 *grs.* of ſilver, how many *lbs.*

Anſwer 3 *lb.*

4th In 17 ingots of ſilver, each 27*oz.* 10*dwts.* how many grains?

Avoirdupoiſe Weight.

1ſt HOW many pounds are there in 256 *Cwt.* 2*qrs.* 17*lb.* of ſugar.

Anſwer 28745 *lb.*

2d In 14048*oz.* how many *cwt?*

Anſwer 7 *Cwt,* 3*qrs.* 10*lb.*

3d How many parcels, each 7 *oz.* are there in 14*lb.* of cloves.

Anſwer 32 parcels.

4th How many ounces and drams are there in 342 common pounds of ſilk? *Anſwer* 5472*oz.* and 87552*drs.*

5th In 914*Cwt.* of lead, how many fodder?

Anſwer 46 *fodder.* 17*Cwt.*

6th In 3 tons of iron, how many *Cwt. qrs.* and *lb.*

Anſwer 60 *Cwt.* 240 *qrs* 6720 *lb.*

Note. The great pound is 24 ounces, and the common pound only 16,

ALTHOUGH the inſtructions already given in money are (I preſume) ſufficient to work every queſtion in Reduction the common way; yet as there is a ſhorter and eaſier method practiſed amongſt merchants, in reducing *C. qrs.* and *lb.* into pounds: I ſhall here ſhew you, how the firſt queſtion in this is perform'd both ways.

Common way 256 2 17

4

1026

28

8225

2052

Anſwer 28745

By the other method 256

256

256

25673

Anſwer 28745

IN order that I may explain the reaſon of the 2d operation more clearly to you, I ſhall place it in the following order.

256

256

256

256|73

28745

NOW it is plain, had I multiplied the number of *Cwts.* by the pounds contained in one hundred, viz. 112; it would have given me the pounds in all the *Cwts.* to which if I had added 73, which is the pounds in 2 *qrs.* 17 *lb.* it would have given the whole number of pounds.

BUT inſtead of multiplying by 112; I only multiply by 111, which being all *ones*, not only ſaves me the trouble of ſetting down my multiplier, but alſo that of multiplying, each product being the ſame. I next find how many pounds, are in 2 qrs. 17 lb. which being 73; I place them to the right hand of the black line.

AND becauſe the difference between multiplying by 111 and 112 will always be the ſame number of pounds as there were hundred weights given to be reduced: I therefore in adding up the ſeveral products, together with the 73, likewiſe add in my Multpilicand, which gives me the anſwer.

Note. When the pounds in the odd quarters and pounds, exceed 99, they could not then be placed to the right hand of your laſt product (as in the above example) but muſt be placed below it, viz. units under units, tens under tens, &c.

Of *Apothecaries* Weight.

1ſt HOW many grains are in 9 *lb.* 7 *oz.* 3 *drs* 1 *ſcr* 14 *grs.* *Anſwer* 55414 *grs.*

2d. How many *lb. oz. dr. ſc. grs.* are there in 74530 *grs.* *Anſwer* 2 *lb.* 11 *oz.* 2 *dr.* 0 *ſc.* 0 *grs.*

3d A chemiſt having 1 *lb.* 9 *oz.* 3 *drs.* 2 *ſc.* 8 *grs.* of powder was minded to make it into medicines of 16 grains each; how many muſt he have? *Anſwer* 664, and 4 *drs.* over.

Of *Long* Meaſure.

1ſt IN 40 yards how many feet, inches, and barley-corns? *Anſwer* 120 *ft.* 1440 *inc.* 4320 *b.c.*

2d In 90 miles how many yards, poles, and furlongs? *Anſwer* 720 *fur.* 28800 *poles.* 158400 *yds.*

3d How many miles and leagues are in 964800 inches? *Anſwer* 5 *lea.* 15 *miles.* and 400 *yards* over.

4th How many barley-corns in a mile? *Anſwer* 190080 *b. c.*

5th How often will a coach wheel, 17 feet 6 inches in circumference, turn round betwixt Newcaſtle and London, being 300 miles? *Anſwer* 90514$\frac{60}{210}$ times

6th How many times oftener will a wheel that is 10 feet 10 inches, turn round; than another that is 19 feet 9 inches; between London and Lincoln, taking the diſtance at 120 miles?
Anſwer 26405 times.

Of *Cloth* Meaſure.

1ſt HOW many quarters and nails, are there in 40 yards of cloth?
Anſwer 160 *qrs.* 640 *nails.*

2d In four pieces of cloth, each 14 yards, how many quarters and nails?
Anſwer 224 *qrs.* 896 *nails.*

3d In 3000 nails, how many Engliſh and Flemiſh ells?
Anſwer 150 *Engliſh ells*, and 250 *Flemiſh ells.*

4th In 47128 nails of Holland, how many pieces, each 12 yards,?
Anſwer 245 *pieces.* 5 *yds.* 2 *qrs.*

5th In 70 bales of Smyrna ſilk, each 11 pieces, each piece 30 yards, how many Engliſh ells, quarters and nails?
Anſwer 18480 *Eng. ells.* 92400 *qrs.* 369600 *nails.*

6th In 17 yards, 1 qr. 2 nails, how many nails?
Anſwer 278 *nails.*

7th In 400 nails, how many yards?
Anſwer 25 *yards.*

Of *Land* Meaſure.

1ſt HOw many roods and perches are there in 20 acre?
Anſwer 80 *roods*, and 3200 *perches.*

2d How many perches are there in 11 acres, 3 roods, 17 poles?
Anſwer 1897 *perches.*

3d In 40 acres, how many roods and perches?
Anſwer 160 *roods.* 6400 *perches.*

4th A gentleman being deſirous to incloſe three fields, the firſt contained 9 acres, and 2 roods; the ſecond 11 acres, 1 rood, and 7 poles; and the third 8 acres, 1 rood, and 12 poles; how many ſhares of 86 perches are contained in the whole?

Anſwer 54 *ſhares*, 15 *perches* over.

5th A farmer bought a field that contained 14 acres, 3 roods, and 7 poles; but he was minded to take an incloſure of 7 acres out of it: how many perches were there in the remainder?

Anſwer 1247 *perches*.

6th One field contains 7 acres, another 10 acres, and a third, 12 acres, 1 rood: how many ſhares of 76 perches each, are contained in the whole?

Anſwer 61 *ſhares*, and 44 *perches* over.

Of *Liquid* Meaſure.

1ſt. IN 46 barrels of ale, how many Gallons?

Anſwer 1472.

2d In 17 gallons, how many quarts and pints?

Anſwer 68 *quarts*. 136 *pints*.

3d How many quarts, pints, and half pints, are in 20 gallons of ale?

Anſwer 80 *qts*. 160 *pints*. and 320 *half pints*.

4th A vintner being deſirous to draw off a pipe of canary into bottles, containing pints, quarts, and 2 quarts, and of each an equal number, how many muſt he have? *Anſwer* 144 of each ſort.

5th In 700 pints of brandy, how many half anchors?

Anſwer 8 *half anchors*, 7 *gals*. and 2 *qts*.

6th How many gallons, quarts, and pints, are there in 7 tun of oil?

Anſwer 1764 *gall*. 7056 *qts* 14112 *pints*.

I told you before how the 16th queſtion in Reduction of money aſcending and deſcending (and all others of the ſame kind) were to be perform'd; I ſhall now demonſtrate to you (by the 4th queſtion above) the reaſon upon which the operation is founded.

IN this queſtion the vintner deſires to have an equal number of pints, quarts, and 2 quart bottles: the quantity therefore which the ſeveral bottles hold, being added together is 7 pints. Now if we ſuppoſe the wine to be drawn off, in bottles which contained 7 pints each, it will then be evident, that as 1 ſeven pint bottle, filled one of each of the other, ſo as many times as the 7 pints are contained in the pipe of wine, juſt ſo many bottles of each ſort he would have.

Of *Dry* Meaſure.

1ſt HOW many buſhels and pecks of wheat are there in 36 quarters?
Anſwer 288 *buſhels*, 1152 *pecks*.

2d How many pecks in 42 chaldron of coals, each 36 buſhels?
Anſwer 6048 *pecks*.

3d In 40 quarters of wheat how many buſhels and pecks?
Anſwer 320 *buſhels*, 1280 *pecks*.

4th How many chaldron of coals are in 8790 pecks?
Anſwer 61 *chaldron*, 1 *buſhel*, 2 *pecks*.

Of *Time*.

1ſt HOW many days in 708560 ſeconds?
Anſwer 8 *days*, 4 *hours*, 49 *minutes*. 20 *ſeconds*.

2d How many Days from *April* 10th to *November* 2d?
Anſwer 205 *days*.

3d Reduce 41 weeks into days, hours, and minutes.
Anſwer 287 *days*, 6888 *hours*, 413280 *minutes*.

4th How many ſeconds in a year, allowing it to be 365 days 6 hours ?
Anſwer 31557600.

Of *Motion*.

The ſun makes his progreſs through the 12 ſigns of the zodiac, in a year's time, each ſign being 30 degrees ; in how many minutes and ſeconds doth he compleat his courſe ?
Anſwer 21600 *minutes*, 1296000 *ſeconds*.

CHAP. III.

Of the RULE of THREE.

Obſervation, 1ſt. } THE Rule of Three is either ſingle or compound.

2d. The ſingle Rule is, when three terms are given to find a fourth.

3d. Of the three terms given ; two of them always imply a ſuppoſition ; and the third is a demand. So this queſtion being propoſed, viz.

If 1 pound of ſugar coſt 4 pence, what will 6 pounds coſt at that rate ?

Here it is plain, that the 1 pound of ſugar, and the 4 pence, are the two terms of the ſuppoſition ;

as

and the 6 pounds of ſugar, is that which demands or aſks the queſtion: and is generally known by following theſe (or ſuch like) words; *How many? How much? What will? How long? How far?* &c.

4th. One of the terms in the ſuppoſition, is always of the ſame name with that which aſks the queſtion: and the other term of the ſuppoſition, of the ſame kind with the 4th term, or anſwer required.

5th The Rule of Three is alſo either *Direct*, or *Inverſe*.

6th. The Rule of Three Direct is, when more requires more, or leſs requires leſs: That is, when (according to the ſenſe and tenor of the queſtion) the 3d term is more or greater than the firſt; and requires the 4th term, more or greater than the 2d in the ſame proportion. Or when the 3d term is leſs than the 1ſt, and requires the 4th term or anſwer leſs than the 2d, in the ſame proportion. For as often as the 1ſt term contains the 2d, or is contained by the 2d; juſt ſo often muſt the 3d contain the 4th, or be contained by the 4th.

7th. Four numbers which are proportionals; the product of the two extremes (which are the 1ſt and 4th) will be equal to the product of the two means, which are the 2d and 3d.

8th. In ſtating all queſtions in this rule, let that term of your ſuppoſition which is of the ſame name with that term of the demand, be your firſt; the other term in your ſuppoſition, which will be of the ſame name with the anſwer required, your ſecond; and conſequently that of your demand, the laſt: and then the queſtion will ſtand in in the following order.

	lb		*d*		*lb*
If	1	:	4	: :	6

To be read thus. If 1 pound coſt 4 pence, what will 6 pound coſt?

And here, according to the ſixth obſervation aforegoing, I find that more requires more: for if the price of one pound be 4 pence: the price of 6 pound will conſequently be ſix times as much.

I ſhall next give you an example, where leſs requires leſs: which may be the following, viz.

If 6 pounds of ſugar coſt 24 pence, what will 1 pound coſt?

		lb		*d*		*lb*
Stated thus,	If	6	:	24	: :	1

IN this queſtion (according to the ſixth obſervation aforeſaid) leſs requires leſs: for if 6 pounds coſt 24 pence, one pound muſt therefore coſt ſix times leſs, viz. 4 pence.

AFTER having ſtated your queſtion as afore directed; to perform the operation, this is the

Rule. Multiply your 2d and 3 terms together, and divide their product by the 1ſt and the Quotient will be the anſwer.

* * *

1ſt If one pound of ſugar coſt four pence, what will 6 pounds coſt?

lb. *d* *lb*

If 1 : 4 :: 6

6

12)24(

2 anſwer.

2d If 6 pound of ſugar coſt 24*d*. what will one coſt?

If 6 : 24 :: 1

1

6)24(

4 anſwer.

3d If 24 pence buy 6 pounds, what will 4 pence buy?

d *lb* *d*

If 24 : 6 :: 4

4

24)24(

1 anſwer.

4th If 3 pound of ſugar coſt 12 pence, what will 6 pounds coſt?

If 3 : 12 :: 6

6

3)72(

24 anſwer.

IN performing the 1ſt queſtion according to the above rule, I multiply my 2d and 3d terms together; and becauſe my firſt term is one (which will neither multiply nor divide) I therefore have the anſwer in pence; which dividing by 12 gives me 2 ſhillings.

AND in the 2d example, my third term being one, for the very ſame reaſon I divide my 2d term by the 1ſt; which likewiſe gives me the anſwer. All the terms in each of the other two examples, viz. the 3d and 4th conſiſting of more Integers than one, takes in the whole general rule, and all of them prove each other.

Note 1ſt. If any of the given terms be a compound one, conſiſting of divers denominations; ſuch term or terms muſt firſt be reduced to the loweſt name mentioned, in order that they may all be ſingle, or whole numbers.

OR if your firſt or third terms (altho' the ſame kind,) viz. both money, both weight, or both meaſure, &c. ſhould be of different denominations, as the one pounds, and the other pence, &c. then you muſt reduce your pounds into pence alſo; and the ſame, if of different denominations of weight, meaſure, &c.

Note 2d When the two terms which are to be multiplied one by the other, will not produce a number large enough to contain your firſt term, or diviſor; you muſt then reduce your 2d or middle term to a leſſer denomination, before you multiply by your third.

Here follow ſome examples for the exerciſe of the learner.

EXAMPLES.

5th. If 1 piece of cloth coſt 3*l* 7*s* 6*d* what will 425 pieces coſt at that rate?
Anſwer 1434*l* 7*s* 6*d*.

6th. If 1 *Cwt*. of ſugar coſt 2*l* 12*s*. what coſt 1*lb*.
Anſwer 5*d* 2*qrs*. and ſomewhat more.

7th. If 1 pair of ſtockings coſt 2*s* 3*d*. what will 25 dozen coſt?
Anſwer 33*l* 15*s*.

8th. If 1 yard of cloth coſt 15*s*. what is the worth o 12 pieces, each 18 yards?
Anſwer £162.

9th. A merchant bought 156 caſks of brandy at 2*l* 15*s* 6*d* per caſk, what did the whole coſt him?
Anſwer 432*l* 18*s*.

[illegible]th. How many quarters of corn for 40 guineas, at 4*s* per buſhel?

Anſwer 26 *qrs*, 2 *buſhels.*

11th. A merchant bought 6 pieces of holland, each 12 ells, for 12*l* 10*s*. what did 1 ell coſt?

Anſwer 3*s* 5*d* 2*qrs*.

12th. A merchant owes 1000 pounds, and compounds with his creditors for 12*s*. 6*d*. in the pound; how much muſt he pay in all?

Anſwer £625.

13th. A goldſmith ſold a tankard for 10*l*. 12*s*. at the rate of 5*s*. 4*d*. per ounce: I demand the weight of it?

Anſwer 39 *oz.* 15 *dwts.*

16th. If a gentleman has an eſtate of 245*l*. 10*s*. a year, how much may be ſpent one day with another, to lay up 60 guineas at the year's end!

Anſwer 10*s*. per day.

17th. At a noble per week how many months board may I have for 50 pound?

Anſwer 37 *months*, 2 *weeks*.

18th. A takes a journey, and travels 40 miles a day. B ſets out the fourth day after him, and travels at the rate of 50 miles a day; I demand in how many days, and after how many miles travel, B will overtake A?

Anſwer he will overtake him in 12 *days*, after having travelled 600 *miles*,

19th. A and B propoſe taking a journey: A. ſets out and travels 40 miles a day, but B cannot ſet out till the fourth day following; I demand how many miles B muſt travel more than A, each day, to overtake him on the 12th?

Anſwer 10 *miles* more each day.

20th. If ſingle rum at 7*s*. 6*d*. per gallon, bears 5 waters to one; what ought I to give for double rum which bears ſeven waters to one, and makes punch of equal quality with the former?

21ſt. If I buy 50 ells of holland for 7*l.* 10*s.* what did it coſt per yard?

Anſwer 4 *ſhillings*, per yard.

I ſhall here perform the operation of the 5th. and 9th. examples aforegoing, in order that the learner may compare them with the 1ſt. and 2d. examples (which are the ſame) in practical Multiplication; and alſo with the ſame queſtions wrought by Practice; that he may know in what particular caſes the work may be ſhortened, by performing the queſtions by any one or other of theſe rules,

Part of Queſtion 5th.	*Part of Queſtion* 9th.
£ *s* *d*	£ *s* *d*
3 7 6	2 15 6
20	20
67.	55.
12	12
810 pence.	666 pence.

Stating of Queſtion 5th.

piece pence pieces
If 1 : 810 : : 425
810

4250
3400

12) 344250 (

2|0 2868|7*s* 6*d*

£4134 7*s.* 6*d*

Stating of Queſtion 9th.

cask pence casks
If 1 : 666 :: 156
156
———
3996
3330
666
———
12)103896(
2|0) 865|8s.
———
£ 432 18s.

THE middle term of each of the above queſtions, conſiſting of divers denominations, I firſt of all reduce them to the loweſt name mentioned, and then having ſtated it properly, work according to the general rule.

S. I think I can now perform many of the aforegoing queſtions according to the general rule given; but as I find in myſelf a certain inquiſitive principle, which will never allow me to take any thing upon truſt; I muſt beg of you, to demonſtrate to me the reaſon on which the general rule is founded; namely, why multiplying the 2d and 3d terms together, and dividing that product by the 1ſt, ſhould give the anſwer required. And alſo why the product of the two means, ſhould be equal to the product of the two extremes, as you told me in the 7th obſervation of this chapter?

M. Both your queſtions, are well deſerving of anſwers; and are, perhaps, of more importance, than you at preſent conceive them to be; as they will not only ſatisfy you with reſpect to the aforegoing opera-

ation:

ations; but by being thoroughly acquainted with the principles on which the rule depends; you may often greatly contract your work: whereas a perſon ignorant of thoſe principles, can never ſtep out of the common road, without being bewildered, and conſequently liable to commit many capital errors.

FIRST then, I deſire that you will once more take a view of the 1ſt and 2d examples of this rule, as they ſtand wrought; and conſider well, what I have there ſaid with reſpect to the reaſonableneſs of the operation.

S. I have conſidered it well; and I am perfectly ſatiſfied with the definition, where the firſt, or laſt term happens to be one unit; becauſe it can neither multiply nor divide; but when any of the ſaid terms conſiſt of more Integers than one, by which (according to the general rule) we can multiply or divide; you will I hope, excuſe me Sir, for telling you that I think the general rule not ſufficiently proved from thoſe examples.

M. I grant you it is not: but as firſt principles, ought always to be clear, ſimple, and ſelf evident; ſo I purpoſely made choice of the 1ſt and 2d examples aforegoing, from which I ſhall draw my arguments to prove the truth of the general rule; which that I may do more clearly, I ſhall here again ſtate and work the 3d example in this rule, in order that you may have it immediately under your eye, without the trouble of turning back to it.

```
     lb   d     lb
If 3  :  12 : : 6
                12
               ----
             3) 72 (
               ----
                24 pence
```

AND here having multiplied and divided as the rule directs, I find the 4th term, or anſwer, to be 24 pence, or 2 ſhillings as before; the reaſon of which is evident: for in this queſtion you are to conſider, that the *6lb.* of ſugar, is multiplied by 12 pence; which is the given price of *3lb.* and conſequently the product muſt be the price of *18lb.* But the price of *6lb.* was only required; wherefore I divide by the 1ſt term 3; for as 6 is the 3d of 18, ſo the 3d part of the price of 18lb. muſt conſequently be the price of 6.

AND hence alſo it neceſſarily follows; that the product of the two means, muſt be equal to the product of the two extremes; which will appear more plain if we place the 4th term or anſwer immediately after the three given terms, as below.

$$\begin{array}{lccccccc} \text{As } 3 & : & 12 & :: & 6 & : & 24 \\ & & 6 & & & & 3 \\ \cline{3-3} \cline{7-7} & & 72 & & & & 72 \end{array}$$

read thus, as 3 is to 12, ſo (is 6 to 24.

HERE you ſee the product of the 1ſt term multiplied by the 4th, is equal to the product of the 2d term, multiplied by the 3d; each being 72; for 3 times the price of *6lb* muſt be equal to 6 times the price of 3 *lb.*

THE following contractions, will ſtill further illuſtrate the general rule; which that the reader may more readily find, I ſhall here give under the title of

Contractions in the *Single Rule* of *Three*, with examples, and a definition of each.

Caſe 1ſt. When the 2d term will divide equally by the 1ſt.

Rule. Multiply the 3d term by the quotient, and the product is the anſwer.

lb *d* *lb*
If 3 :)12(: : 6
4
4
24 anſwer

Definition. HERE having ſtated the queſtion as before; I divide the middle term 12 pence (which is the given price of 3*lb.*) by 3, which muſt give the price of 1*lb.* And then the queſtion is exactly reduced to the 1ſt propoſition, viz. If 1*lb.* : 4*d.* : : 9*lb.* The reaſon of which operation having been already fully proved, the other muſt be undeniably true.

Caſe 2. When the 3d term, will divide equally by the 1ſt.

Rule. Multiply the 2 term by the quotient, and the product will be the anſwer.

d *lb* *d*
Example. If 12 : 3 :)24(
2
2
6 anſwer.

Definition. THE reaſon of this contraction, is ſelf evident; for if the price of 3*lb.* viz 12 pence is contained twice in 24 pence : conſequently 24 pence muſt buy twice 3*lb.* It therefore having been already proved (by the 1ſt and 2d practical queſtions in Diviſion) that whether we multiply 24 by 3; and divide that product by 12 : or divide 24 by 12, and multiply the quotient by 3, it muſt produce the ſame effect; I think nothing further need be ſaid to prove the truth of the general rule.

Caſe 3d. When the 1ſt term can be divided equally by the 2d.

Rule. Divide the 3d term by that quotient, and the laſt quotient will be the anſwer.

$$\begin{array}{lccccc} & \overset{d}{\underline{24}} & : & \overset{lb}{6} & :: & \overset{d}{\underline{)4(}} \\ & 4 & & & & 1lb \end{array}$$

Example. If 24 : 6 : :)4(

Definition. Here having divided my firſt term (which is the price of 6*lb*) by ſix, the quotient will be the price of 1*lb*, viz. 4 pence; which being equal to my 3d term, (it being alſo 4 pence) conſequently the the anſwer mnſt be 1*lb*.

Caſe 4. When the firſt term can be equally divided by the third.

Rule. Divide the middle, or 2d term by the quotient, and the laſt quotient will be the anſwer.

Example, If $\overset{d}{24} : \overset{lb}{6} :: \overset{d}{8}$ wrought thus $8)\underline{24|}\ \ 3)\underline{6|}$

$\qquad 3 \quad 2$

Definition. Here having divided my firſt term by my laſt; I find the quotient to be 3. If then 8 be contained in the price of 6*lb.* 3 times: conſequently ſo often as 3 is contain'd in 6*lb.* juſt ſo many pounds will 8*d* buy.

Note 3d. When the given terms happen to be ſuch, as don't immediately come under any of the 4 aforementioned caſes; the work may yet be often abreviated by

Caſe 5th. When you can ſpy any number which will divide your 1ſt. and 3d, or 1ſt and 2d terms equally.

Rule. Take the two quotients, inſtead of ſuch terms; and then work according to the general rule: except you can again abreviate your work by any of the aforemention'd 4 caſes; as in the following.

$$\begin{array}{lccccc} & lb & & d & & lb \\ \textit{Example.}\ \text{If} & 9 & : & 12 & :: & 24 \\ & 3 & & & & 8 \end{array}$$

In this example you ſee, none of the firſt 4 caſes could take place. I therefore according to Caſe 5th ſeek for a number which will divide the 1ſt and laſt terms equally, which I find to be 3. The quotient thence ariſing will be 3 and 8, as you ſee (which according to the definition given of diviſion in page 60) will bear the ſame proportion to each other, as did the two terms firſt given, viz. 9 and 24. therefore taking 3 and 8 for my firſt and laſt terms, inſtead of 9 and 24; the queſtion will then ſtand thus.

$$\begin{array}{lccccc} & lb & & d & & lb \\ \text{If} & 3 & : & 12 & :: & 8 \end{array}$$

which being again abreviated according to the rule in *Caſe* 1ſt, reduces it to this propoſition, If 1*lb* : 4*d* :: 8*d* which has already been fully proved.

Caſe 6th. When your firſt and laſt terms do not exceed twelve, and your middle term happens to conſiſt of divers denominations,

Rule. You may then (without reducing your middle term to the loweſt denomination) multiply by your third term as in practical multiplication, and divide the product by your firſt term in the ſame manner. If your firſt and third terms exceed 12. Then multiply and divide by any two figures which multiplied together will produce any of the ſaid terms.

ALTHO' I have now, I hope, fully demonſtrated to you, the reaſon on which the general rule is founded, for working all queſtions in the Rule of Three Direct; and alſo given you a general rule for ſtating of them: yet there may be many queſtions, propoſed in this rule, which require ſome diſcernment to find the proportion (when it is not poſitively expreſs'd) which one thing bears to another. Now I remember you told me, that you thought you could work any of the foregoing queſtions; let me ſee, how you would ſtate, and work the 20th queſtion viz. this. If ſingle rum at 7*s.* and 6*d.* per gallon bear 5 waters to 1; what ought I to give for double rum which bears 7 water to 1, and makes punch of equal quality with the former?

```
                      wat.  s d    wat
S. In this manner  If 5  :  7 6 : : 7
                            12
                           ----
                            90
                             7
                          -----
                        5) 630 (
                          -----
                       12) 126 (
                          -----
                            10s. 6d. per gal.
```

M. Here (as I indeed foreſaw) you have miſtaken the proportion: for before you can aſcertain it, you muſt add 1 gallon of ſingle rum, to the 5 gallons of water; and a gallon of double rum to the 7 gallons.

Then the punch being of equal quality, the proportion will be this.

gal *s* *d* *gal*
If 6 : 7 6 : : 8
12

90
8

6) 720 (

12) 120 (

10 ſhillings per gallon.

S. This ſeems to me a diſtinction without a difference; for the water being of equal quality, how can the proportion be altered, by adding one to each? for as each term is increaſed one; I think they ought to have the ſame proportion to each other, as the firſt given terms had.

M. Impoſſible; for altho' any two terms being multiplied, or divided, by one and the ſame number; their products or quotients, will have the ſame reaſon to each other as the two firſt given numbers had; yet adding 1 to each greatly alters the property; for 6 cannot bear the ſame proportion to 8, as 5 does to 6, which I thus prove. Suppoſe the 1ſt term to have been 2, and the laſt 4, here you ſee 2 is exactly the half of 4, and conſequently if 2 galls. was worth 7*s*. 6*d*. 4 muſt be worth 15*s*. which is double the ſum: But if we add 1 to each; the 1ſt term is then 3, and the laſt 5, conſequently 3 being more than half of 5, they cannot have the ſame proportion.

TO prove this operation right, let us ſee what each gallon of punch coſt: and 1ſt when we have added 1 gallon of ſingle rum at 7*s.* 6*d.* per gallon, to 5 gallons of water, we have 6 gallons of punch for that money; which is juſt 1*s.* 3*d.* per gallon. And when we have likewiſe added 1 gallon of double rum, to the 7 gallons water; we then have 8 gallons of punch for 10*s* which is alſo 1*s.* 3*d.* per gallon; which being both of equal price and quality, the operation muſt of conſequence be right

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Of the RULE of THREE, *Inverſe.*

Obſervation 1ſt. THE Rule of Three Inverſe is, when more requires leſs, or leſs requires more. That is, when (according to the ſenſe and tenor of the queſtion) the 3d term is more or greater than the 1ſt, and requires the 4th term or anſwer leſs than the 2d in the ſame proportion. Or when the 3d term is leſs than the 1ſt, and requires the 4th term, or anſwer, greater than the 2d in the ſame proportion. For as often as the 3d contains the 1ſt, or is contain'd by the 1ſt; ſo often muſt the 4th contain the 2d, or be contained by the 2d. So this queſtion being propoſed, viz. (If 1 man can perform a piece of work in 12 days, in what time will 6 men perform the ſame?)

It is plain that more requires leſs; for common reaſon tells us, that if 1 man can perform it in 12 days; 6 men will perform it in the 6th part of the time. Therefore we cannot ſay here, as in direct proportion; as 1 is to 12, ſo is 6 to a 4th number, which ought (in that caſe) to be 6 times as much as 12; but contrariwiſe by an inverted proportion; as 1 is to 12, ſo is 6 to a fourth number, which (in this caſe) muſt be 6 times leſs.

By the ſame rule; in the following queſtion leſs requires more. If 6 men can do a piece of work in 2 days, in what time will 1 man perform the ſame? For it is evident, that 1 man will take 6 times the time that 6 men would do, to perform the ſame work.

Obſervation 2d. Hence it follows, that in all queſtions in the ſingle Rule of three Inverſe, the product of the 3d term multiplied by the 4th; ſhall be equal to the product of the 1ſt multiplied by the 2d.

IN ſtating queſtions in this rule, the ſame directions are to be obſerved, as have already been given in the Rule of three Direct; which when you have done and reduced your terms, if need be, according to the note in this chapter, to find the anſwer, obſerve the following

Rule. Multiply the 1ſt and 2d terms together, and divide the product by the 3d term, and the quotient will give the 4th term or anſwer required.

EXAMPLES.

IF 3 men can perform a piece of work in 12 days, in what time will 6 men perform the ſame?

m *days* *m*
If 3 : 12 : : 6
3
6) 36 (
6 anſwer

IF 6 men can perform a piece of work in 6 days, in what time would 3 men perform the ſame?

m *days* *m*
If 6 : 6 : : 3
6
3) 36 (
12 anſwer.

HERE after having ſtated it according to the rule, I then multiply my 2d term by my 1ſt, and dividing the product by my 3d term, I have 6 days for the anſwer.

IN the 1ſt of thoſe queſtions, more require leſs; and in the laſt, leſs require more; and mutually prove each other.

AND here according to the 2d obſervation on this rule, we ſhall find that the product of the 3d and 4th terms; are equal to the product of the 1ſt and 2d, as will better appear by ranging the 1ſt of the above queſtions in the following manner.

$$\begin{array}{lccccccc} \text{As } 3 & : & 12 & :: & 6 & : & 6 \\ & & 3 & & 6 & & \\ \hline & & 36 & & 36 & & \end{array}$$

Note That all queſtions in the ſingle Rule of Three Inverſe, may be made direct; by making the 1ſt term, the 3d; and the 3d, the 1ſt.

Here follow more Examples for the exerciſe of the Learner.

3d How many yards of ſhalloon that is ell wide, will line 140 yards of cloth that is 3 quarters wide?

Anſwer 84 *yards.*

4th How many yards of matting, that is half yard wide; will cover a room that is 18 feet wide, and 30 feet long?

Anſwer 128 *yards.*

5th Suppoſe one lend 1200 pounds, 12 months; how much muſt be lent 72 months to requite the former favour.

Anſwer 200 *pounds.*

6th How much in length, that is 3 inches broad, will make a ſquare foot?

Anſwer 48 *inches.*

7th Suppoſe a building raiſed in 16 months by 240 men, and another of the ſame model was required in 4 months, how many men muſt be employed to compleat the ſame?

Anſwer 960 *men.*

7th If when the price of a buſhel of wheat is 6*s.* 3*d.* the penny loaf weighs 9*oz.* what muſt the penny loaf weigh when wheat is 4*s.* 6*d.* per buſhel?

Anſwer 12*oz.* 10*dwts.*

6th If a field will graze 24 horſes 9 weeks, how many horſes will it graze 56 weeks?

Anſwer 6 *horſes.*

10th Suppoſe 800 ſoldiers were placed in a garriſon, and their proviſions were computed ſufficient for 2 months; how many ſoldiers muſt depart that the proviſions may ſerve them 5 months.

Anſwer 480 *men.*

11th A ciſtern has 3 cocks, which will empty it in 48 hours; how many cocks of the ſame bore will empty it in 6 hours.

Anſwer 24 *cocks.*

12th A piece of tapeſtry is 3 ells flemiſh wide, and 4 ells flemiſh long, and it is required to be lined with ſomething that is but 3 qrs. of a yard wide; I demand how many yards there muſt be to compleat the lining?

Anſwer 9 *yards.*

13th If when beer is 74 ſhillings per butt, 24 ſhillings worth, will ſerve 288 men; how many men will the ſame 24 ſhillings worth ſerve, when beer is 5*l.* 8*s.* per butt.

Anſwer 197 *men.*

14th If a perſon performs a journey of 908 miles in 12 days; when the days are 16 hours long: how many days will it require of him when they are but 12 hours long?

Anſwer 16 *days*.

IN order that I may demonſtrate to you the reaſon on which the general rule is founded, for working the ſingle Rule of Three Inverſe; I ſhall here ſtate and work the 4th example aforegoing.

FIRST then, having reduced my 1ſt and 3d terms to the ſame name, viz. half yards (for I could not reduce my half yard, into feet) I then ſtate it in this manner. If 12*H,yds.* : 30*ft l.* :: 1*h,yd?* read thus; If 12 half yards in width require 30 feet in length; how many feet in length, will 1 half yard in width require?

Scho. I do not rightly underſtand this queſtion. Had the breadth and length, of any piece of ſtuff been given which would have covered the rooom; and it had been required how much in length, of any other kind of ſtuff of a different breadth, would have been neceſſary for the ſame purpoſe; I could then have form'd the ſtating: altho' the reaſon of the operation would have ſtill remained a myſtery to me.

Maſt. Tis true, the breadth and length of a piece of cloth which would have covered the room is not given, tho' plainly imply'd; for I think it is pretty clear, that a piece of cloth of the ſame breadth and length of the room would have covered it; and therefore it is the ſame, as if the queſtion had been propoſed in this manner, viz. If a piece of cloth which is 18 feet, or 12 half yards broad, and 30 feet long, will cover a room; I demand how much in length, of a ſtuff which is but half a yard broad, will anſwer the ſame purpoſe?

Scho. I now clearly underſtand it, and therefore beg you will proceed to ſhew me how the operation is perform'd, and the reaſon on which it is founded.

h. yds. ft. h. yd.
Maſt. Thus, If 12 : 30 :: 1
12

3)360 *Anſwer.* 360 *feet.*

120

HERE it is evident, that if it requires a piece 30 feet, or 6 yards in length, which is 18 feet, or 12 half yards broad, that it muſt require more in length of that ſtuff which is but half a yard in breadth. And therefore as half a yard is one twelfth of the given breadth, ſo the length of the ſtuff required muſt be twelve times the length of the piece given; and hence it follows, that the product of the 3d and 4th, muſt be equal to the product of the 1ſt and 2d, as will appear by ranging them in a direct line, thus, 12 : 30 :: 1 : 360

| *For 360 feet multiplied by one half yard, muſt be equal to 30 feet multiplied by 12 half yards, as appears by the work.* | 12 : 30 :: 1 : 360
12 1
360 360 |

To make this yet more plain, let us ſee how many pieces of 30 feet long each, are contained in 360 feet, which we will find to be; 12 conſequently laying thoſe 12 pieces which are each ½ yard wide, one by the ſide of another, it will make a piece of 30 feet in length, and 12 half yards in breadth, which is exactly equal to the given piece, and fully proves the truth of the operation.

As therefore the reaſon why the general rule will hold good in all caſes (where any of the given terms conſiſts of more Integers than one) are the ſame with thoſe I have already given in the Rule of Three Direct, I think it needleſs to repeat them again here.

CHAP. IV.

Of the DOUBLE RULE of THREE;

Perform'd by two Single Rules.

Obſervation 1ſt. } IN the double Rule of Three, (whether Direct or Inverſe,) there are always 5 terms given to find a 6th, the 3 firſt of which imply a ſuppoſition, and the 2 laſt make a demand.

2d One of the terms in the ſuppoſition is always of the ſame name or kind, with the 6th term or anſwer required; and the other two of the ſame name with thoſe in the demand.

3d. It is called the Double rule of Three Direct, when each of the ſingle rules by which it is performed look for a 4th term in a Direct proportion. 4

4th In ſtating all queſtions in this rule; let that term in your ſuppoſition which is of the ſame name or kind with the anſwer required, be your 2d term in the 1ſt ſtating: one of the other terms in your ſuppoſition (no matter which) be your 1ſt; and your 3d, that I in the demand which is of the ſame kind of your 1ſt. Having ranged your terms in this order, and perform'd the operation according to the rule given, for working the Single rule of Three Direct; reſerve your 4th term or anſwer for your 2d or middle term in your 2d ſtating; making your other term in your ſuppoſition your 1ſt, and the other term of your demand, (which will be of the ſame name with your 1ſt,) your 3d or laſt term: and then working again as before, the 4th term ſo found, will be the anſwer required.

Note 1ſt. The ſame is to be obſerved here, with reſpect to reducing your terms if of different denominations, to one denomination, as in the Single Rule of Three.

For an Example let us take the following.

Exam. 1ſt If 4 men in 3 months ſpend 25*l.* how much will 8 men ſpend in 9 months?

men £ *men*
1ſt ſtating If 4 : : 25 : : 8
8

4) 200 (

50 firſt anſwer,
2d ſtating If 3 : 50 : : 9
9

3) 450 (

£ 150 anſwer.

Defini-

Firſt then having obſerved that the 25*l.* is that term in my ſuppoſition which is of the ſame name or kind with the anſwer required (for it is money that is demanded) I therefore make it the 2d or middle term in my 1ſt ſtating; the 4 men, which is another term of my ſuppoſition, I make my 1ſt term; and the 8 men being of the ſame kind with my 1ſt, I make my 3d or laſt term, and then the proportion will be this, viz.

If 4 men ſpend 25*l.* in a certain time, (viz. 3 months,) what will 8 men ſpend in the ſame time; which according to the rule of Three Direct; I find to be 50*l.* I next proceed to my 2d ſtating; ſaying if 3 months require 50 for a certain number of men, (viz 8) how much will it require to ſerve them 9 months; which being likewiſe Direct, I work as before, and find the 4th term or anſwer to be 150*l.*

Note 2d. If I had taken 3 months, and 9 months for my 1ſt and laſt terms of my 1ſt ſtating: and 4 men and 8 men for the firſt and laſt terms of my 2d ſtating, it would have given the ſame anſwer.

2d. If 8 reapers have 3*l.* 4*s.* for 4 days work; how much will 48 reapers have for 16 days work?
Anſwer 76*l.* 16*s.*

3d. If 10 buſhels of oats ſerve 18 horſes 20 days; how many buſhels will ſerve 60 horſes 36 days?
Anſwer 60 *buſhels.*

4th. If 12 *Cwt.* be carried 80 miles for 4*l.* for how much might 7 *Cwt.* be carried 50 miles at that rate?

1ſt ſtating,
Cwt. £ *Cwt.*
If 12 : 4 :: 7
7
12)28(2
24
4
20
12)80(6
72
8
12
12)96(8
96
..

2d ſtating
M £ *s* *d* *m*
If 80 : 2 6 8 :: 50
20
46
12
560
50
8|0)2800|0
12)350(
2|0)2|9 - 2
£ 1-9-2

Of the Double *Rule* of *Three*, *Inverſe*.

Obſerva-tion 1ſt. THE double Rule of Three Inverſe is, when one, or both of the ſingle rules by which it is perform'd, requires a fourth term in an inverted proportion.

2d. In ſtating all queſtions in the double Rule of Three Inverſe the ſame Inſtructions are to be obſerved, as thoſe already given in the double rule direct.

3d. When therefore you have ſtated your queſtion, and conſidered (according to the ſenſe and tenor of it) whether it looks for a fourth term in a direct or inverted proportion, proceed then to perform the operation

peration as in either of thoſe caſes directed, according to which you find it belongs.

Scho. I thought both ſtatings could never prove Inverſe.

Maſt. All authors I have ever ſeen on this ſubject affirm they cannot; but I hope I ſhall be able clearly to prove the contrary by the third example following.

If 9 buſhels of oats ſerve 8 horſes 12 days, how many days will 24 buſhels ſerve 16 horſes ?

1ſt ſtating.	2d ſtating.
h. *d.* *h.*	*b.* *d.* *b.*
If 8 : 12 : : 16	If 9 : 6 : : 24
8	6
16) 96 (6	9) 144 (
96	16 anſ.

AND here again for ranging the terms in their due order, I firſt obſerve amongſt the terms of the ſuppoſition, which of them are of the ſame denomination with the term required, which I find to be 12 days: for it is required how many days 24 buſhels will ſerve 16 horſes. I therefore make the ſaid 12 days my 2d term in the 1ſt ſtating: I next take 8 horſes, which is another term of the ſuppoſition for my 1ſt term; and laſt of all, I place that term in the demand which is of the ſame name with my 1ſt term, for my 3d: and then it will read thus.

If 8 horſes in 12 days conſume a certain quantity of oats, viz. 9 buſhels; In how many days will 16 horſes eat the ſame quantity?

AND here, according to the ſenſe and tenor of the queſtion, I find the proportion to be Inverſe: that is more requires leſs; for 16 horſes being equal to twice 8, ſo conſequently they will eat the ſame quantity of oats in half the time that 8 horſes did: I therefore work as directed in the Single rule Inverſe, and find that 16 horſes would eat 9 buſhels of oats in 6 days; as appears by the operation.

WHEREFORE reſerving this anſwer for the 2d term in my next ſtating; I then ſay, if 9 buſhels (which is the other term in the ſuppoſition) ſerve a certain number of horſes, (viz. 16,) 6 days; how many days will 24 buſhels, which is the other term of the demand, ſerve them? Which proportion being direct, I work as that rule requires, and find the anſwer to be 16 days.

Queſt. 2d. If 7 men can reap 24 acres of wheat in [illegible] days; how many men can reap 100 acres in 5 days?

1ſt ſtating.	2d ſtating.
acres *m* *acre.*	*day* *m* *days*
If 84 : 7 : : 100	If 12 : 8 : : 5
7	12
84) 700 (8	5) 96 (
672	19 *men*
28 over	

Note. The anſwer to this queſtion ſhould have been 20 men, but on account of the large remainder in the 1ſt ſtating, which being alſo men, and conſequently could not be reduced: it therefore could not be wrought perfectly by 2 ſtatings, without the help of fractions.

Queſt. If 240 men can perform a certain piece of work in 12 days, when the day is 12 hours long; in how many days will 720 men perform the ſame, when the day is 16 hours long?

1ſt ſtating.	2d ſtating
M. *days* *men*	*hrs.* *days* *hrs.*
If 240 : 12 :: 720	If 12 : 4 :: 16.
12	12
72|0)288|0(4	16)48(3 Anſ.
288	48
...	..

HENCE it is plain that both ſtatings may be Inverſe; for according to the 1ſt ſtating; If 120 men can perform a certain piece of work in 12 days, when the day is 12 hours long, 720 men muſt do it in fewer days, when the day is the ſame length, viz, in 4 days Again in the 2d ſtating it is evident, that if when the day is 12 hours long a certain piece of work can [illegible] done by a certain number of men (viz 720) in 4 days, that it will require fewer days for the ſame number of men to perform the ſame work, when the day is 16 hours long: and therefore in both ſtatings more requires leſs.

I ſhall here propoſe the laſt queſtion again in a different manner: by which you will ſee, that notwithſtanding all the given numbers in it are exactly the ſame, yet, by changing the name or denomination of two of them, viz. thoſe of *men*, to thoſe of *miles*; the ſenſe of the queſtion is entirely altered, and the anſwer conſequently very different from that of the laſt: as the one ſtating will here be Direct, and the other Inverſe: whereas the two ſtatings in the laſt queſtion were both Inverſe.

Queſt. 4th. If a footman travels 240 miles in 12 days, when the days are 12 hours long: in how many days may he travel 720 miles, when the day is 16 hours long?

1ſt ſtating	2d ſtating
mls. *days* *mls.*	*hrs.* *days* *hrs.*
If 240 : 12 :: 720	If 12 : 36 :: 16
12	12
24|0)864|0(36	16(432(27 Anſ.
72	32
144	112
144	112
...	...

HERE you ſee that notwithſtanding the terms in the 1ſt ſtating, are the ſame with thoſe in the 1ſt ſtating of queſtion 3d; yet by making the 1ſt and third terms *miles*, which were *men* in the other, the ſenſe of the queſtion (as I obſerv'd before) is entirely alter'd; and becomes direct, which in the laſt queſtion was Inverſe. For if 240 miles can be travelled in 12 days, when the days are a certain length; it is plain that it muſt require more days of the ſame length, to travel 720 miles in: and conſequently the 4th term or anſwer of the 1ſt ſtating of each, (which are very different one from the other) being made the middle term of the 2d ſtating, (they being both Inverſe) muſt give very different anſwers; the one being only 3 days, and the other 27; as appears by the work.

ALTHO' the above method of working the double rule of three, both Direct and Inverſe, is generally taught; yet I would by no means recommend the practice of it: as in many caſes where the anſwer to the firſt ſtating happens to be of divers denominations, as in the 4th example in the double rule Direct, it renders the operation both long, and tedious; as you are obliged in the 2d ſtating, to reduce them to the loweſt denomination mention'd; and in others again, where the 2d or middle term of the firſt ſtating happens to be men, horſes, ſhips, &c. as in the 2d example in this rule, which are incapaple of being divided into parts; conſequently if there is any remainder, except you know the uſe of fractions, it is thrown away in the 2d ſtating; which cauſes the anſwer to be imperfect: as I ſhall prove by working the ſame queſtion at one ſtating in the rule of five. However I thought it neceſſary to give you a few examples to work by this method, as the reaſon of the operation appearing ſo very clear, I might more eaſily, from it, prove the propriety of the rule I ſhall give to work the ſame examples by one ſtating, viz. The double Rule of Three compounded of five numbers.

The *Double Rule* of THREE, *Compounded*, Of five Numbers, performed by one Stating.

THIS Rule, commonly called the rule of five, though little inſiſted on by moſt of our arithmeticians, yet is not only the moſt commodious, but alſo the moſt exact way of performing the Double rule of Three, for the reaſons before given, and which will clearly appear by comparing the following examples with thoſe in The Double rule Direct and Inverſe, perform'd by two ſeveral ſtatings.

Obſervation 1ſt. In ſtating your queſtions, let your terms be ranged in ſuch order, that your 1ſt and 4th may be of 1 kind, and your 2d and 5th of another; and your 3d or middle term of the ſame name with the 6th term or anſwer required : to do which, 1ſt obſerve amongſt the 3 terms of your ſuppoſition, which of them is of the ſame name or kind with the anſwer, and reſerve that for your 3d term. The other 2 terms of your ſuppoſition make your 1ſt and 2d, no matter which of them ſtand 1ſt. Having placed thoſe 3 terms in their proper order, make your 4th the ſame as the 1ſt, and the 5th the ſame with the 2d.

2d Conſider your 1ſt, 3d and 4th terms as a ſtating in the Single Rule of Three, neglecting your 2d and 5th; and note well, whether it is Direct or Inverſe.

3d. THEN neglecting your 1ſt and 4th, and conſidering your 2d, 3d and 5th, as the 2d ſtating : diſcover in like manner whether it be Inverſe or Direct, and then work as directed in the following caſes.

Caſe 1ſt. If when both prove to be Direct Proportion.

Rule. Multiply your 3d, 4th, and 5th terms together, and divide their product by the product of the 1ſt and 2d, and the quotient will be the anſwer to the queſtion.

Caſe 2d. When the 1ſt proves Direct, and the laſt Inverſe.

Rule. Multiply the 2d, 3d, and 4th together, for a Dividend, and your 1ſt and laſt for a Diviſor.

Caſe 3d. When the 1ſt proves Inverſe, and the laſt Direct.

Rule. Multiply your 1ſt, 3d and 5th together, for a Dividend; and your 2d and 4th together, for a diviſor, and the quotient will be the anſwer.

Caſe 4th. When both ſtatings are Inverſe.

Rule. Multiply the 3 firſt terms for a dividend, and the 2 laſt for a Diviſor, and the quotient will be the anſwer.

SO the 1ſt example in the Double Rule of Three, being again propounded, viz.

IF 4 men ſpend 25 pounds in 3 months; I demand how many pounds will ſerve 8 men, 9 months? The terms would be ranged.

$$\text{Thus If } \overset{\textit{men}}{4} : \overset{\textit{mo}}{3} :: \overset{£}{25} : \overset{\textit{men}}{8} : \overset{\textit{mo}}{9}$$

$$\text{Or thus If } \overset{\textit{mo}}{3} : \overset{\textit{me}}{4} : \overset{£}{25} : \overset{\textit{mo}}{9} : \overset{\textit{me}}{8}$$

And firſt I proceed (according to the 1ſt & 2d obſervation in this rule) to diſcover whether the queſtion be Direct or Inverſe: ſaying, if 4 men ſpend 25*l.* in a certain time (viz. 3 months) how much will 8 men ſpend in the ſame time? Which I find to be Direct; for more men muſt ſpend more money in the ſame time.

I next examine the 2d ſtating in the ſame manner; ſaying, If in 3 months 25*l.* be ſpent by a certain number of men (meaning 4) how many will be ſpent in 9 months by the ſame number of men? which I find likewiſe to be Direct; for the ſame number of men will ſpend more money in 9 months, than they could do in 3 months. Wherefore according to the rule in caſe 1ſt, I multiply my three laſt terms together for a dividend, and my 2 laſt terms together for a diviſor, and the quotient gives the anſwer. See the operation.

men *mo* £ *men* *mo*
If 4 : 3 : 25 : : 8 : 9
3 8
12 200
9
12) 1800 (
150£

Here note, that which ever of the 2 terms in your ſuppoſition (which are of the ſame kind with the 2 in the demand) you place 1ſt or 2d, it makes no difference in the manner of working, provided they be both Direct.

Definition. The terms of the above queſtion being ranged either according to the 1ſt or 2d poſition, the reaſon of the rule given for working it, will appear evident from the following conſiderations.

1ſt If 4 men in 3 months ſpend a certain ſum of money, then it is plain that 12 men will ſpend the ſame in 1 month. For the very ſame reaſon then, if 8 men ſpend a certain ſum in 9 months, then will 72 men ſpend the ſame ſum in 1 month, or 1 man in 72 months. Let us ſuppoſe then 12 men and 72 men, (for it is no matter whether men or months) here their reſpective times being reduced to a time common to both, viz. 1 month, it is thereby cancelled in both, and the proportion is then as follows; which being reduced to a queſtion in the Single rule of Three Direct; I perform the operation accordingly, and find the anſwer as before to be 150 pounds.

If 12 : 25 : : 72
25
———
360
144
———
12) 1800 (
———
150£

FROM the foregoing principles is alſo deduced the general rule for working the Double Rule of Three Direct, viz. When you have with 3 of your terms formed the 1ſt ſtating (according to the rule given for working by 2 ſingle ſtatings) then ſet the other term of your ſuppoſition under yonr 1ſt term, and the other term of your demand under your 3d, and then the work will ſtand thus.

Men £ *Men*
If 4 : 25 : : 8
3 *months* 9 *months*

Then having multiplied your 4 men by 3 months, and your 8 men by 9 months, your queſtion is again reduced to this propoſition *viz.*

Men £ *Men*
If 12 : 25 : : 72

HENCE the reaſon of the 1ſt operation is fully proved; for the terms which are multiplied together for a Dividend, and alſo thoſe for a Diviſor, being the ſame in both; conſequently, they muſt produce the ſame anſwer.

N. B. *This definition alſo accounts for the reaſon of the Rule for working* Double Fellowſhip, *where the partner's ſeveral ſtocks have reſpect to their particular time.*

S. Will this rule which you have given for working the double Rule of Three Direct, likewiſe hold good for working the Double Rule Inverſe?

M. When both ſtatings are *Inverſe*, the only difference will be, that when you have multiplied thoſe terms together which ſtand under each other: then as the quotient looks for a fourth term in an inverted proportion, you muſt multiply the 1ſt product, by the middle term for a Dividend; whereas if it had been *Direct*, you would have multiplied the laſt product, by your middle term, & the product of your firſt two would have been your Diviſor. Whereas in the Double Rule Direct, the product of your third term, and that which ſtands under it, is multiplied by your middle term for a Dividend.

BUT when one of your ſtatings proves Direct, and the other Inverſe, altho' the manner of placing your terms will be the ſame; yet the rule for working the queſtion will differ, according to the choice you make of your firſt and third terms, for your firſt ſtating; as I ſhall prove by the firſt queſtion in the Double Rule Inverſe: but firſt, I ſhall ſhew how the ſaid queſtion is to be performed by one ſingle ſtating. The queſtion was this.

If 9 buſhels of oats ſerve 8 horſes 12 days; how many days, will 24 buſhels ſerve 16 horſes?

HAVING then 1ſt ranged my terms in their proper order, according to the 1ſt obſervation in this rule, they will,

buſh. horſes days buſh. horſes
Stand thus If 9 : 8 : 12 : : 24 : 16

horſes buſh. days horſes buſh
Or thus If 8 : 9 : 12 : : 16 : 24

I then proceed to find whether the queſtion be Direct or Inverſe in this manner. Saying (according to the 1ſt method of ranging them) If 9 buſhels in 12 days will be eat by a certain number of horſes; (viz. 8) in how many days will 24 buſhels be eat by the ſame number of horſes? which firſt ſtating I find to be direct; for more buſhels, muſt require more days for the ſame number of horſes to eat them in. I next compare the 2d ſtating in the ſame manner: ſaying again, If 8 horſes in 12 days eat a certain quantity of corn (9 buſhels) in how many days will 16 horſes conſume the ſame quantity? which 2d ſtating I find to be Inverſe: for 16 horſes muſt eat the ſame quantity of corn, in leſs time than 8 horſes did: wherefore according to the rule for working caſe 2d in this chapter, I multiply my 3 middle terms together for a dividend, and my firſt and laſt together for a diviſor, and dividing the one by the other, I have the anſwer.

BUT the terms being ranged according to the 2d method; the firſt ſtating will be found Inverſe, and the laſt Direct. And therefore according to the rule in caſe 3d, I multiply my 1ſt, 3d and 5th terms together for a dividend, and my 2d and 4th terms for a diviſor, and dividing the one by the other I have the ſame anſwer as before. See the operation both ways.

According to the 1ſt method of ranging them. Thus,

b. *b.* *d.* *b.* *b.*
If 9 : 8 : 12 :: 24 : 16
16 12
144 288
8
144)2304(16*ds.*
144
864
864
...

And according to the ſecond method. Thus,

b. *b.* *ds.* *b.* *b.*
If 8 : 9 : 12 :: 16 : 24
16 12
144 288
8
144)2304(16ds.
144
864
864
...

HERE you may obſerve, that notwithſtanding the order in which the terms are ranged, differ from each other; yet by the rule given to work each of them, the ſame number are multiplied together for a dividend, and alſo the ſame together for a diviſor, and conſequently muſt give the ſame anſwer. See alſo the ſame queſtion perform'd by two ſingle ſtatings in the Double Rule of Three Direct. I ſhall

I ſhall now ſhew you how the method of working will differ (according to the terms you make choice of for your 1ſt and 3d) when it is ſtated in this manner, viz.

$$\begin{array}{lccccc} & & b & & d & & b \\ \text{1ſt method,} & \text{If} & 9 & : & 12 & :: & 24 \\ & & 8 & & & & 16 \end{array}$$

HAVING firſt ranged my terms as before directed, (according to this method of ſtating) viz. that term of my ſuppoſition which is of the ſame name with the anſwer required, in the middle; one of my other terms in my ſuppoſition for my 1ſt: and that one of the demand (which is of the ſame name with my firſt) my 3d or laſt term; I next place my other term of ſuppoſition regularly under my firſt, and alſo the other term of demand under my third; which two are likewiſe of the ſame kind, viz. both horſes. I next proceed to diſcover whether the firſt ſtating be Direct or Inverſe; ſaying, If 9 buſhels be eaten in 12 days by a certain number of horſes, (viz. 8) in how many days will 24 buſhels be eaten by the ſame number?

THIS clearly appearing to be Direct; let us then ſuppoſe the work to be proſecuted accordingly, viz the 2d and 3d terms multiplied together; and that product divided by the 1ſt. Now the quotient ariſing from that, would (according to the method for working the ſaid queſtion by 2 ſingle ſtatings) have been the middle term in your 2d ſtating. Let us therefore then again, ſuppoſe ſuch quotient (whatever it be) to be placed between 8 and 16, and ſee whether this laſt ſtating be Direct or Inverſe: ſaying, if 8 horſes in a certain time (viz. the laſt quotient) eat a certain quantity of oats: in what time will 16 horſes eat the ſame? which laſt ſtating, I find to be Inverſe; for 16 horſes muſt eat the ſame quantity of oats in leſs time, than 8 horſes would have done. Therefore (according to the rule for working the Single Rule of Three Inverſe) I ſhould multiply my 1ſt term 8, by the ſuppoſed middle term, and divide that product by 16, and the quotient would have been the anſwer.

BUT as we only ſuppoſed the 1ſt ſtating to have been performed; therefore the anſwer (which ought to have been the middle term, in the 2d ſtating, and by which the 1ſt. ſhould be multiplied) is an unknown ber.

NOW here note well, that whether I multiply the 12 and 24 together, and divide that product by 9, and again multiply 8 by that quotient, and divide that product by 16. I ſay whether I do that, or multiply my 24 by 12, and that product again by 8, and divide the laſt product by the product of 16 multiplied by 9, it is the ſame thing

Definition. For that number of the 1ſt ſtating, by which I ſhould divide, being 9: and that by which I divide in the 2d ſtating, 16; it is then plain, that if I were to divide the product (of 24 multiplied by 12) by 9: the quotient would be 9 times leſs than the ſaid product; conſequently therefore if I multiply the 9th part of that product by 8, and divide by 16; it muſt give a quotient equal to the whole product multiplied by 8, and divided by 9 times 16.

THIS being proved, the operation will be as follows.

buſh.

```
       buſh     d      buſh
       If 9  :  12  :  24
          8             16
         ——            ———
         192           144
          12
        ————
144) 2304 (16
      144
     ————
       864
       864
     —————
```

IN the above operation I multiply 24 by my 8, and then that product again by 12, which laſt product is 2034 for a dividend. I next multiply my 9 by 16, the product of which is 144 for a diviſor; and laſtly dividing the one by the other, I have 16 days for the anſwer.

BUT if for my 1ſt ſtating I take the other term in my ſuppoſition for my 1ſt, and the other in the demand for my 3d term; then it would be Inverſe, and the 2d ſtating Direct, and conſequently the way of working it would differ, as you will ſee by the operation.

```
      b       d         b
   If 8   :  12   : :  16
      9                24
     ———              ———
     144              288
                        8
                     ————
              144) 2304 (16
                    144
                   ————
                    864
                    864
                   ————
```

THE principles on which this operation is founded, having then been made plain; we may eaſily deduce from thence, the reaſon of the general rule given for working all queſtions in the Double Rule of Three, both Direct, and Inverſe, by one ſingle ſtating. For by obſerving (according to the laſt method of ſtating; what terms are to be multiplied together for a dividend, and which for a diviſor) according to that ſtating in which the Inverſe proportion is found, I mean whether the 1ſt or 2d; and obſerving alſo at the ſame time the manner in which the terms ſtand ranked in both, it will be eaſy to diſcover what particular place, [illegible] term by the laſt way of ſtating, muſt poſſeſs in the former; and conſequently the very ſame terms muſt be multiplied together in each.

Here follow ſome more Queſtions for the exerciſe of the Learner.

If 30*s.* be the hire of 8 men for 3 days: how many days muſt 20 men work for 15*l*?

Anſwer 12 days.

4th If 56*lb.* of bread will be ſufficient for 7 men, 14 days: how much bread will ſerve 21 men 3 days?

Anſwer 36*lb.*

5th If 7 men can reap 84 acres of wheat in 12 days, how many men will reap 100 acres in 5 days?

Anſwer 20 *men.*

6th An uſurer put out 156 pounds, to receive intereſt for the ſame: and when it had continued 8 months, he received for principle and intereſt 161*l.* 4*s.* I demand at what rate per cent per annum, he received intereſt?

Anſwer 5*l. per cent.*

8th If 240 men can perform a certain piece of work in 12 days, when the days are 12 hours long: in how many days will 720 men perform the ſame, when the days are 16 hours long?

Anſwer 3 days.

SECTION 1ſt.

How to diſcover by Inſpection (only) whether your Queſtion be *Direct* or *inverſe*.

WHEN a queſtion in the Double Rule of Three is propoſed, you may eaſily diſcover by inſpection (only) whether it be Direct or Inverſe, by obſerving the following

Directions. Firſt range your terms in 1 line, without having any other regard to the order of their places, than making that term of your ſuppoſition which is of the ſame name with the anſwer required, your middle term; placing the other 2 terms of your ſuppoſition to the left hand of it; and the 2 terms in your demand, on the right.

THIS done, obſerve well amongſt the terms of your ſuppoſition, what reſpect one; has to another: ſuch as the 1 being days, and the other hours; time, and men; horſes, &c. If the 1ſt and 2d have ſuch reſpect one to the other, then your queſtion is Direct: but if your 2 terms which have ſuch relation one to the other, be your 1ſt and 3d, or 2d, and 3d: then the queſtion is Inverſe. Without having the leaſt regard then whether the Inverſe proportion falls to be in the 1ſt or 2d compariſon, you may work any queſtion in the Double

Rule of Three, either Direct or Inverſe, by obſerving this general rule.

Rule. Multiply your 2 terms in your ſuppoſition (ſo connected) one by the other, and that product again by that term in your demand, which is of the ſame name with your other term in your ſuppoſition, for a Dividend; and the other 2 terms, multiply together for a Diviſor, and the quotient will be the anſwer.

EXAMPLE.

IF 15 men can perform 240 yards of trenching in 21 days: In how many days will 45 men perform 480 yards of trenching?

```
        yds     m.n     days         men     yds
    If  240  :  15   :  21    : :    45   :  480
         45             15
        ----           ------
       1200             105
       960               21
      ------           ------
      10800             315
                        480
                       ------
                       25200
                      1260
                    --------
             108|00) 1512|00 (14 anſwer
                     108
                    --------
                      432
                      432
                     -----
                      ...
```

HERE you ſee that my 2d and 3d terms in my ſuppoſition, (viz. 21 days, and 16 men) are thoſe which have ſuch relation to each other as befote deſcribed; I therefore multiply them together, and that product again by 480, being that term in the demand which is of a different name from any of the other 2, viz. yards; and then dividing that product, by the product of the other term in the ſuppoſition, multiplied by the other term in the demand, gives 14 for the quotient, which is the anſwer to the queſtion.

If all the terms in your ſuppoſition bear ſuch relation one to the other as already deſcribed; Then both your ſtatings are Inverſe; to perform which, this is the rule. Multiply your three terms in the ſuppoſition one into another for a dividend, and your two terms in the demand for a diviſor; and laſtly dividing one by the other, the quotient will be the anſwer. For a proof of this, let us again take the 3d example in the double rule Inverſe, which being ſtated as directed in the 1ſt ſection in this rule, will ſtand thus, If 240*m.* : 12*h.* 8*d.* :: 480*m.* : 16*h.* Here you ſee men, hours, and days have all relation one to another; and therefore according to the rule already given, muſt be multiply'd together for a dividend, and the other two terms for your diviſor; the reaſon of which you will find explain'd in the definition given of the 1ſt queſtion in this rule. See the whole work.

men *h.* *d* *men* *h.*
If 240 : 12 : 8 :: 480 : 16
12 16

2880 2880
8 480

7680)23040(3 days 7680
23040

BEFORE I quit this rule, I ſhall give you one example more, to prove that queſtions may be very exactly perform'd by one ſtating, which (if wrought by two ſingle ſtatings) would give very imperfect anſwers; for which purpoſe I ſhall again propoſe the 5th example in the double rule of Three Inverſe, viz.

If 7 men can reap 84 acres of wheat in 12 days; how many men will reap 100 acres in 5 days?

Acs. dys. me. acs. dys.
See the operation. If 84 : 12 : 7 :: 100 : 5
5 7
420 700
12
420)8400(20 *days*.
840
...0

See the ſame queſtion wrought by two ſingle ſtatings in the double rule of Three Inverſe, Page 110.

CHAP. V.

Rules of PRACTICE by *Aliquot Parts*.

THIS is no more than a contraction of the Rule of Three: for when the price of 1 Integer, (viz. 1 yard, ell, hundred weight, &c.) is given; and the price of any quantity of integers are required, the anſwer may more readily be found, by taking the equal parts of a penny, a ſhilling or a pound (according to the table given in Reduction) as the nature of the queſtion requires. That I may not therefore confound the learner, or burthen his memory with unneceſſary rules: I ſhall give all ſuch queſtions, where the price of an integer is any of thoſe equal parts of a penny, ſhilling, or pound, under

SECTION 1ſt.

WHEN the given price of 1 Integer happens to be any of thoſe equal parts of a penny, ſhilling, or pound, as you find expreſſed in the tables given in Reduction.

Rule. Divide the number of Integers given, by ſuch parts, and the quotient will be ſo many pence, ſhillings, or pounds, as you take parts for: and the remainder (if any) will be ſo many times the price of 1 Integer.

Definition. This is no more than what you have already been taught in Reduction of Money: for if you conſider the number of Integers whoſe price is required, to be ſo many farthings, pence, ſhillings, three and four-pences, &c: or any other equal part, which the price of the given integers happens to be: it becomes exactly a queſtion in Reduction: and is the ſame as if it had been demanded in ſo many farthings, pence, ſhillings, three and four-pences, &c. how many pound?

Note. When the given price of an Integer is 2*d*, 3*d*, 4*d*, 6*d* or 8*d*, and the number of Integers whoſe price are required exceeds the parts of a pound for which they ſtand in table 3d. Then you may (if you pleaſe) divide by ſuch part, and your quotient will give you the price in pounds at once, and the remainder (if any) will be the price of 1 Integer.

EXAMPLES.

1ſt At 1 farthing per yard, what will 4800 yards coſt?
Anſwer 5£

2d At 1 penny per yard, what will 900 yards coſt?
Anſwer 3*l.* 15*s.*

3d At 1 ½*d* yard, what will 2135 yards coſt?
Anſwer 13*l.* 6*s* 10½*d*

4th At 2 pence per yard, what will 725 yards coſt?
Anſwer 6*l.* 0*s.* 10*d.*

5th At 3 pence per yard, what will 486 yards coſt?
Anſwer 6*l.* 1*s.* 6*d.*

6th At 4 pence per yard what 573 will yards coſt?
Anſwer 9*l.* 11*s.*

7th At 6 pence per yard, what will 1690 yards coſt?
Anſwer 42*l.* 5*s.*

8th At 1*s.* 8*d.* per yard, what will 287 yards coſt?
Anſwer 23*l.* 18*s.* 4*d.*

9th At 2*s.* per yard, what will 354 yards coſt?
Anſwer 35*l.* 8*s.*

10th At 2*s.* 6*d.* per yard, what will 5424 yards coſt?
Anſwer 678*l.*

11th At 3*s.* 4*d.* per yard, what will 769 yards coſt?
Anſwer 128*l.* 3*s.* 4*d.*

12th At 4*s.* per *lb.* what will 238 *lb*, coſt?
Anſwer 47*l.* 15*s.*

13th At 5*s.* per ſtone, what will 342 ſtone coſt?
Anſwer 85*l.* 10*s.*

14th At 6*s* 8*d.* per buſhel, what will 1540 buſhels coſt?
Anſwer 573*l.* 6*s.* 8*d.*

SECTION 2d.

WHEN the price of an Integer is no aliquot part of a penny, or ſhilling.

Rule. Firſt take ſome part of it that is an aliquot part; and for the remaining part of the price, take it out of the foregoing part or parts, and then adding the quotients together, you have the anſwer in pence or ſhillings, according to which of them it was you took parts of, which being divided again by 12 and 20 will give you the anſwer in pounds.

Note 1ſt. The remainder will always be of the ſame denomination with your dividend.

1ſt. 4712 *yds.* at $\frac{3}{4}$ pr *yd.*	2d. 5780*lb* at $2\frac{1}{4}$ pr *lb.*
| $\frac{1}{2}$ | $\frac{1}{2}$ |)4712	| 2 | $\frac{1}{6}$ |)5780
	d
| $\frac{1}{4}$ | $\frac{1}{2}$ | 2356	| $\frac{1}{4}$ | $\frac{1}{8}$ | 963(4
1178	120·5
12)3534	2|0(108|3·9
d	
2|0)29|4 6	£5 4 3·9
£14·14·6	

IN performing the firſt queſtion, I firſt take $\frac{1}{2}$ penny for $\frac{1}{2}$ of 1 penny, and conſidering the queſtion to be the ſame, as if it had demanded how many pence were in 4712 half-pennies? I therefore divide by 2, and the quotient is 2356 pence. But I ſtill want 4712 farthings, and becauſe $\frac{1}{4}$ *qrs.* is $\frac{1}{2}$ of 2 farthings I therefore take half of the price at 2 farthings, viz. half of 2356 pence, which is 1178 pence; which being added together, and divided by 12 and 20, gives the anſwer 14*l.* 14*s.* 6*d.*

THE 2d queſtion I perform in this manner. Firſt I ſeek what 5780*lb.* would coſt at two pence per *lb.*

and conſidering that it would juſt coſt ſo many twopences; I therefore divide by 6, ſaying 2*d.* is ⅙ of a ſhilling, which gives me 963 ſhillings for the quotient, and 2 over, which 2, is 2 pences, or 4 peace. I ſtill want the price of 5780*lb.* at a farthing, and becauſe 1 farthing is ⅛ of 2*d.* I therefore take ⅛ of 963*s.* 4*d* which is 120*s.* 5*d.* which being added together and divided by 20 give me 54*l.* 3*s* 9*d* for the anſwer.

Note 2d. In the 1ſt queſtion. If inſtead of taking half the price of 2 farthings, for the price of one farthing; I had ſaid ¼ is ¼ of a penny and divided the number of yards again by 4, it would have been the ſame.

Qu. 3d. 6128 at 3*d* ¼
Anſwer 12*l.* 13*s.* 9*d.*

4th. 7812 at 3*d* ¾
Anſwer 122*l.* 1*s.* 3*d.*

5th. 6001 at 4*d* ½
Anſwer 112*l.* 10*s.* 4*d.* ½

6th. 6100 at 5*d* ¾
Anſwer 146*l* 2*s.* 11*d.*

7th. 1218 at 6*d* ½
Anſwer 32*l.* 19*s.* 9*d.*

8th. 7101 at 7*d.*
Anſwer 207*l.* 2*s.* 3*d.*

9th 6120 at 7*d* ¾
Anſwer 197*l.* 12*s.* 6*d.*

10th. 6100 at 8*d* ¼
Anſwer 209*l.* 13*s.* 9*d.*

11th. 8000 at 8*d* ½
Anſwer 283*l.* 6*s.* 8*d.*

12th. 6000 at 8*d* ¾
Anſwer 218*l.* 15*s.*

13th. 9000 at 9*d.*
Anſwer 337*l.* 10*s.*

14th 5918 at 9*d* ¾
Anſwer 240*l.* 8*s.* 4*d.* ½

15th. 8121 at 10*d.*
Anſwer 338*l.* 7*s.* 6*d.*

16th. 1002 at 10*d* ½
Anſwer 43*l.* 16*s.* 9*d.*

17th. 4680 at 10*d* ¾
Anſwer 209*l.* 12*s.* 6*d.*

18th. 1260 at 11*d.*
Anſwer 57*l.* 15*s.*

19th. 6121 at 11*d.* ¼
Anſwer 286*l.* 18*s.* 5*d.* ¼

20th. 1234 at 11*d* ½
Anſwer 59*l.* 2*s.* 7*d.*

Note. When the price of an Integer is 10 pence; you may annex a cypher to the number of Integers whoſe price are required, and divide by 12 and 20.

THE reaſon of this operation is plain; for adding a cypher is the ſame as multiplying by 10; ſo that it brings it all into pence, which being divided by 12 and 20, gives the anſwer in pounds.

SECTION 3d.

WHEN the price of an Integer is more than a ſhilling, and leſs than 2 ſhillings.

Rule. Let the given quantity ſtand for ſhillings; and take parts for the odd pence, or pence and farthings, (which are above a ſhilling) out of the given number, which being added together, the total will be the anſwer in ſhillings.

EXAMPLES.

¼ is ¼) 486 at 12¼*d*

12) 121 (½

10 1 ½
486

2|0) 49|6 1 ½

£ 24 16 1 ½

2d 1216 at 12*d*. ¾
Anſwer 64*l*. 12*s*.

3d 6121 at 16*d*.
Anſwer 331*l*. 11*s*. 1*d*.

4th 1612 at 15*d* ¼
Anſwer 102*l*. 8*s*. 7*d*.

5th 7612 at 15*d*. ¾
Anſwer 499*l*. 10*s*. 9*d*.

6th 6100 at 16*d*.
Anſwer 406*l*. 13*s*. 4*d*.

7th 23 0 at 17*d* ½
Anſwer 170*l*. 12*s*. 6*d*.

8th 4560 at 18*d*.
Anſwer 342*l*.

9th 9876 at 19*d*. ½
Anſwer 802*l*. 8*s*. 6*d*.

10th 6543 at 20*d*. ½
Anſwer 558*l*. 17*s*. 7*d*. ½

11th 5432 at 20*d*. ¾
Anſwer 469*l*. 12*s*. 10*d*.

12th 4321 at 21*d*.
Anſwer 378*l*. 1*s*/ 9*d*.

13th 2100 at 21*d*. ½
Anſwer 188*l*. 2*s*. 6*d*.

14th 1090 at 22*d*.
Anſwer 99*l*. 18*s*. 4*d*.

15th 9010 at 22*d*. ¼
Anſwer 835*l*. 14*s*. 0*d*. ½

19th 6812 at 22*d*. ¼
Anſwer 645*l*. 14*s*. 5*d*.

IN performing the 1ſt queſtion, I let the given number ſtand for ſhillings. Then I divide the ſaid given number by 4, and by 12; which gives me the price at ¼, in ſhillings: and becauſe I cannot ſo conveniently add that line to the given number, which (as I ſaid before) I let ſtand for ſhillings; I therefore bring it down, and ſetting it below my other line, add them together, and divide by 20; which gives me the anſwer in pounds.

Note 3d When the price of an Integer is 22 pence, you may annex a cypher to the given number, and divide by 12, (as at 10*d*.) then adding both together, the total will be the anſwer in ſhillings, which dividing by 20, gives the anſwer in pounds.

THE reaſon of this operation is likewiſe plain, for adding a cypher, and dividing by 12 (according to *Note* 1ſt in this ſection) giving you the price at 10*d*. in ſhillings; therefore adding that to the number of integers firſt given (which was the price at ſhillings) which being divided by 20, gives pounds.

Note 4th Any of the foregoing queſtions in the ſection, where the price of an Integer is any of thoſe equal parts of a pound that you find in your table; you may perform the operation either according to this ſection, or ſection 1ſt.

SECTION 4th.

WHEN the given price of an Integer is any number of ſhillings under 20, which are no aliquot parts of a pound.

Rule. Multiply the given quantity by half the price, and double the 1ſt figure in the product for ſhillings, and the reſt of the product will be pounds,

Definition. Here it is plain, that if I had multiplied the whole number of Integers, by the whole price, and divided the product by 20, the anſwer would have been pounds; conſequently then, if I multiply the ſaid number of Integers by half the price, and divide by 10, which is alſo half of 20; it muſt be the ſame thing. For it has already been proved (in Diviſion) that to divide by 10, is no more than to cut off a figure from the right hand of your Dividend; therefore the remaining part of your Dividend muſt likewiſe be pounds. Now as the figure cut off from your Dividend, is not divided at all; therefore we are to conſider it as multiplied by the whole price, which is the reaſon why we double it for ſhillings.

EXAMPLES.

1ſt 7618 at 6*s*.
3
———
2285*l*. 8*s*.

2d 191 at 8*s*.
Anſwer 76*l*. 8*s*.

3d 171 at 14*s*.
Anſwer 119*l*. 14*s*.

4th 171 at 16*s*.
Anſwer 136*l*. 16*s*.

5th 712 at 18*s*.
Anſwer 640*l*. 16*s*.

IN performing the 1ſt queſtion in this ſection, I multiply the given integers by 3, which is half the price of 1 integer; ſaying 3 times 8 is 24, which 4 I double for ſhillings, (which being 8 I ſet down to the right hand, and ſo proceed, carrying my 2 to the next figure, as in common Multiplication, and the product is pounds.

SECTION 5th.

WHEN the given price of an Integer is any odd number of ſhillings under 20, and yet none of thoſe alliquot parts of a pound which you find expreſſed in your table.

Rule. Multiply the given integers by the price, and that product divide by 20, and the quotient will be the anſwer in pounds, &c.

EXAMPLES.

1ſt 121 at 3*s*.
Anſwer 18*l*. 3*s*.

2d 860 at 7*s*.
Anſwer 301*l*.

3d 612 at 9*s*.
Anſwer 275*l*. 8*s*.

4th 121 at 11*s*.
Anſwer 66*l*. 11*s*.

5th 600 at 13*s*.
Anſwer 390*l*.

6th 190 at 15*s*.
Anſwer 142*l*. 10*s*.

SECTION 6th.

WHEN the price of an Integer is ſhillings and pence, and yet no alliquot part of a pound.

Or, if ſhillings, pence, and farthings.

Rule. Multiply the given quantity of Integers by the ſhillings and take parts for the reſt, which being added together, the ſum will be the anſwer in ſhillings.

Note. If the given price be ſuch as you can divide into equal parts of a pound, as 7*s.* 6*d.* then you may take 5*s.* for a fourth; and again 2*s.* 6*d.* for the half of 5*s.* and the anſwer will be pounds.

EXAMPLES.

yds.
3*d.* $\frac{1}{4}$ | 126 at 9*s.* 3*d.*
9

1134
31 6

2|0) 116|5 6

£58-5-6

2d 86*lb.* at 6*s* 10*d* per *lb.*
Anſwer 29*l.* 7*s.* 6*d.*

3d 10 *cwt.* at 12*s.* 4*d.*
Anſwer 6*l.* 3*s.* 4*d.*

4th 30 ſtone at 4*s.* 9*d.*
Anſwer 7*l.* 2*s.* 6*d.*

5th 73 at 7*s.* 6*d.*
Anſwer 27*l.* 7*s.* 6*d.*

6th 70 at 7*s.* 4*d.* $\frac{3}{4}$ per dozen.
Anſwer 25*l.* 17*s.* 8*d.* $\frac{1}{2}$

7th 12, at 13*s.* 10*d.* $\frac{1}{2}$
Anſwer 8*l.* 6*s.* 8*d.*

IN performing the 1ſt queſtion in this ſection, I firſt multiply the given Integers by the ſhillings; and becauſe 3*d.* is $\frac{1}{4}$ of a ſhilling, I divide the given integers by 4, which gives me 31 ſhillings, and 2 over;

which 2, is 2 three-pences, or ſix-pence. I then add my two lines of ſhillings together, and divide the ſum by 20, which gives me the anſwer.

SECTION 7th.

WHEN the given price of an Integer is pounds, and ſhillings.

Rule. Multiply the Integers given, by the pounds; then proceed with the ſhillings if an even number, according to ſection 4th: but if the ſhillings be odd, then according to ſection 5th; and adding them together the total will be the anſwer.

EXAMPLES.

1ſt 26 at 4*l.* 8*s.*
4
104
10 8
£ 114 8*s.*

2d 49 at 3*l.* 7*s.*
7
2|0) 34|3 (
17 3
147
£164 3

3d 36 at 5*l.* 13*s.*
Anſwer 203*l.* 8*s.*

4th 48 at 7*l.* 10*s.*
Anſwer 360*l.*

5th 26 at 11*l.* 14*s.*
Anſwer 304*l.* 4*s.*

6th 15 at 4*l.* 13*s.*
Anſwer 69*l.* 15*s.*

7th 17 at 9*l.* 15*s.*
Anſwer 165*l.* 15*s.*

8th 16 at 3*l.* 6*s.*
Anſwer 52*l.* 16*s.*

9th 64 at 4*l* 7*s.*
Anſwer 278*l.* 8*s.*

THE operation of the 1ſt and 2d queſtions in this ſection will appear ſo clear by reviewing the 4th and 5th ſection; that they need no further explanation.

SECTION 8th.

WHEN the given price of an Integer is pounds, ſhillings, and pence.

Rule. If the ſhillings and pence be the alliquot parts of a pound; divide the given Integers by ſuch alliquot parts, and the quotient will be pounds; then multiply the given Integers by the pounds which one Integer coſt, and adding them together, you will have the ſum required.

EXAMPLES.

1ſt. 47 at 3*l*. 3*s*. 4*d*
Anſwer 148*l*. 16*s*. 8*d*.

2d. 20 at 4*l*. 13*s*. 4*d*.
Anſwer 93*l*. 6*s*. 8*d*.

3d. 17 at 2*l*. 6*s*. 8*d*
Anſwer 39*l*. 13*s*. 4*d*.

4th. 30 at 1*l*. 2*s* 6*d*.
Anſwer 33*l*. 15*s*.

2d. If the ſhillings and pence be not the alliquot parts of a pound, or if there be ſhillings, pence and farthings given with the pounds; then reduce the pounds and ſhillings into ſhillings, and multiply the given Integers by the ſaid ſhillings: next take parts with the reſt of the price, and add them together as before.

EXAMPLES.

```
1ſt. 3d½ | 120 at 4l. 7s. 3d½
              87
            -----
           10440
½ is ⅙ |      30
               5
            -----
      2|0)1047|5
            -----
       £523 - 15s. Anſ.
```

2d. 14 *Cwt* at 2*l* 10*s*. 6*d*.
Anſwer 35*l* 7*s*.

3d 21 at 5*l* 14*s*. 7*d*¼
Anſwer 120*l*. 6*s*. 8 ¼

4th. 70 at 1*l*. 14*s*. 7*d*.
Anſwer 121*l*. 0*s*. 10*d*.

5th. 46 at 3*l*. 19*s*. 8*d*¼
Anſwer 183*l*. 6*s*. 7*d*.

6th. What coſt 156 caſks of brandy at 2*l.* 15*s.* 8*d.* per caſk?

```
 d                              £   s   d
| 4 | 1/3 |  - - 156             2  15   8
| 4 | 1/3 |       55            20
             ------          ----
                 780            55
                780
             ------
               8580
                 52
                 52
             ------
           2|0)868|4
             ------
           £434 - 4s.
```

7th. What is the value of 425 pieces of cloth at 3*l.* 7*s.* 6*d.* per piece?

```
   d                            £   s   d
| 6 | 1/2 |  - - 425            3   7   6
                  67           20
              ------         ----
                2975           67
               2550
              ------
               28475
                 212
              ------
           2|0)2868|7
              ------
           £1434 - 7s. - 6d.
```

IN performing the 1ſt queſtion; I firſt reduce the pounds and ſhillings into ſhillings; by which multiplying the number of integers, I have the price of the whole, at 87 ſhillings I next want the price of the whole Integers at 3 pence; wherefore I ſay 3*d* is $\frac{1}{4}$, therefore 120 being divided by 4, gives me 30 ſhillings more. Then for the price at $\frac{1}{2}$*d.* which yet remains to be taken; (becauſe $\frac{1}{2}$ is $\frac{1}{6}$ of 3 pence) I divide my 30 ſhillings, which is the price at 3 pence, by 6, which gives the price at $\frac{1}{2}$ penny: and laſtly, adding thoſe 3 lines together, and dividing by 20, I have the whole price.

Note 1ſt. The operation of the 6th and 7th examples aforegoing, being compared with thoſe of the 5th and 9th in the Rule of Three (page 90) and alſo with the 1ſt and 2d in the 1ſt ſection of *Compound Multiplication*, will I hope, prove the general rule given (in page 51) to know when by any one or other of thoſe rules, a queſtion may be conveniently formed.

Note 2d. The 9th queſtion in the Rule of Three will not agree with the ſame queſtion in *Compound Multiplication*, and in Practice. As I have (there by miſtake) made the price of an Integer 2*l.* 15*s.* 6*d.* inſtead of 2*l.* 15*s.* 8*d.* but that can make no difference with reſpect to your judging of the length of the operation in each rule.

SECTION 9th.

WHEN the price of an Integer, and the given quantity happen both to be of different denominations.

Rule. Find the price of the whole Integers, as you do in *Compound Multiplication* and for the remaining parts of an Integer, take a part or parts of the given price of 1 Integer.

EXAM

EXAMPLES.

cwt	qrs	lb		£	s	d	
12	2	14	of tobacco, at	3	14	0	per cwt.
			Anſ. 46 14 3				
17	0	19	of Sugar - - -	2	2	6	per d°
			Anſ. 38 1 6¾				
4	1	16	of ſoap - - - -	3	12	0	per d°
			Anſ. 15 16 3½				
10	0	12	of tallow - - -	1	19	6	per d°
			Anſ. 19 9 2½				
5	1	0	of tobacco - - -	2	17	0	per d°
			Anſ. 14 9 3				
4	3	0	of ſugar - - - -	2	18	6	per d°
			Anſ. 13 17 10½				
7	0	19	of ſugar, - - -	3	16	9	per d°
			Anſ. 27 4 11				
5	2	10	of tobacco, - - -	2	18	6½	d°
			Anſ. 16 7 2½				
7	1	14	of tobacco, . - -	3	15	9¼	d°
			Anſ. 27 18 9½				
9	2	26	of tallow, - - -	4	10	9¼	d°
			Anſ. 43 19 6				
54	3	24	of tobacco, - - -	2	15	8	per d°

* * *

£ s d.

		£	s	d
qr.	½	2	15	8
2				6
		16	14	0
				9
r		150	6	0
1.	½	1	7	10
lb.				
14	½	0	13	11
7	½	0	6	11½
2	1/7	0	3	5¾
1	½	0	1	11¾
		0	0	11¾
		£153	1	1¾

I ſhall here explain the manner in which this laſt queſtion is performed; which if you make yourſelf maſter of, you will find no difficulty in working any queſtion that can be propoſed in Practice.

FIRST then, I find the price of the Integers as in *Compound Multiplication*; I next want the price of 3 qrs. which being no equal part of 1cwt. I therefore 1ſt find the price of 2 qrs, ſaying 2 qrs. is ½; where dividing the price of 1cwt. by 2, I have the price of ½ cwt. Then as ¼ is ½ of, ½ cwt. I ſay again, ½ is ¼ ſetting it oppoſite to the price of 2 qrs. which having divided by 2, I have the price of ¼ cwt. Again 14lb. being ½ of 1 qr. I therefore divide the half of 1 qr. by

[illegible]

2; again, 14lb being $\frac{1}{2}$ of 1 qr. I therefore divide the price of a qr. by 2, and I have the price of 14lb. then for 7lb. I again take $\frac{1}{2}$ of the laſt line; but I ſtill want the price of 3lb more: now as 3lb. is no aliquot part of any of the parts already taken, I therefore take 2lb. firſt, and 2lb. being $\frac{1}{7}$ of 14lb, I therefore divide the price of 14lb. viz. 13*s.* 11*d.* by 7, which gives me the price of 2lb. the half of which is the price of 1lb. which being all added together gives the price of the whole.

S. I obſerve throughout the whole examples, which you have given in Practice, the given price is always that of 1 integer; I ſhould therefore be glad to know whether or not, queſtions could be performed by this rule, when the given price is that of more Integers than one.

M. In all queſtions in the Single Rule of Three Direct, where the 1ſt and middle terms are whole quantities; and the middle term will divide equally by the 1ſt, then that quotient being the price of 1 Integer, conſequently it is then reduced to a queſtion in Practice.

SECTION 10th.

TARE and TRETT.

AS we may naturally ſuppoſe, that every perſon in the grocery-way, muſt be acquainted with the common allowances made for *tare*, *trett*, *cloff*, *ſuttle*, &c. It would therefore be thought ſuperfluous to ſhew (under a different title) what would in reality be no more than Reduction over again. I however thought proper to give it a place according to the common order in which it generally follows; leaſt the learner ſhould think I had forgot it, and to tell him that I paſs it over, to make room for other rules, which are of more importance to him.

CHAP. VI.

Of INTEREST.

THIS Rule (with every other branch which it comprehends, viz. *Factorage*, *Brokerage*, *Inſurance*, *&c.*) depends on the very ſame principles with the RULE of THREE; and is no more, than ſo many different methods of propoſing the ſame queſtion. However, that I might be as diſtinct and clear as poſſible, I thought proper to treat of it in a Chapter by itſelf; as there are many uſeful contractions in *Intereſt*, &c. which are not common to every queſtion, in the Rule of Three.

Intereſt is either SIMPLE or COMPOUND.

SIMPLE Intereſt is the premium allowed by the borrower to the lender, upon the principal only, for a certain time propoſed.

In *Intereſt* three things are principally to be conſidered, 1ſt. The PRINCIPAL, or ſum lent.

2d. The TIME, for which it is lent.

3d. The RATE per CENT, which is a certain ſum of money agreed on between the borrower and lender, for the uſe of every hundred pounds for a year, or 12 months.

SECTION 1ſt.

To find the Intereſt of any given ſum, for a year.

Rule. Multiply the Principal by the rate per Cent, and divide that product by 100; and the quotient is the Intereſt required.

Note, If the Intereſt be required for more years than one; Then multiply the Intereſt for one year, by the number of years; and the product will be the anſwer.

EXAMPLES.

1ſt. If 100*l* in one year give 5*l* Intereſt, what will 972*l* give at the ſame rate?

Anſwer 48*l* 12*s*.

$$\begin{array}{r}
\pounds \\
972 \\
5 \\
\hline
48|60 \\
20 \\
\hline
12|00
\end{array}$$

By the Rule of Three this queſtion would have been ſtated thus. If 100*l* : 5*l* : 972*l*. and the operation exactly the ſame: for to divide by 100, is no more than to cut off two figures from the right hand of your dividend, and thoſe to the left are pounds. Then multiplying thoſe cut off, by 20, and again cutting off two figures as before, gives you the ſhillings, &c.

2d. What is the intereſt of 500 pounds for one year at 4*l* per cent per annum?

Anſwer 20*l*.

3d. What is the intereſt of 65 pounds for 2 years, at 5*l* per cent per annum?

Anſwer 6*l*. 10*s*.

4th. What is the intereſt of 650 pounds for 5 years, at 6*l* per cent per annum?

Anſwer 195*l*.

5th. What is the amount of 612 pounds for 10 years, at 6*l* per cent per annum?

Anſwer 979*l*. 3*s* 11*d*. $\frac{3}{4}$

To find the intereſt of any ſum for $\frac{1}{4}$, $\frac{1}{2}$ or $\frac{3}{4}$ of a year, beſides the number of years given in the queſtion.

SECTION 2d.

Rule. For $\frac{1}{4}$ take a fourth part of the intereſt for one year; for $\frac{1}{2}$ a year, take half of the intereſt for one year, and for $\frac{3}{4}$ of a year, take the intereſt for $\frac{1}{4}$ and $\frac{1}{2}$ of a year added together; which being added to the intereſt for the reſt of the time, gives the intereſt for the whole time required.

EXAMPLES.

1ſt What is the intereſt of 240*l*. for 3 years and $\frac{3}{4}$ at 5 per cent per annum?

240 | $\frac{1}{2}$ | $\frac{1}{2}$ | 12*l* Intereſt for 1 year.
5   3

for 1 *year* 12|00   30 d° for 3 years
| $\frac{1}{4}$ | $\frac{1}{2}$ | 6 d° for $\frac{1}{2}$ year
3 d° for $\frac{1}{4}$ year

£45

Here after having found the intereſt for 1 year according to ſection 1ſt, I mlutiply the ſaid intereſt by 3 which gives me the intereſt for 3 years; then by Practice, I take ½ the intereſt, for 1 year for the intereſt of half year, and again half of that for a quarter, which having added together I have 45*l.* for the anſwer.

2d What is the intereſt of 468*l.* 12*s.* 4*d.* for one year and ¾, at 6 per cent per annum?
Anſwer 49*l.* 4*s.* 1*d.*

3d. What is the intereſt of 112*l.* 10*s.* 4*d.* for 5 years and ½, at 6 per cent per annum?
Anſwer 37*l.* 2*s* 6.

4th. What is the Intereſt of 468*l* for 4 years and a quarter, at 6 per Cent per annum?
Anſwer 119*l.* 6*s.* 8*d* ¼

5th What is the intereſt of 1000*l.* for 2 years and ¾ at 4 per cent per annum? *Anſwer* 110*l.*

SECTION 3d.

TO find the commiſſion, brokage, inſurance, &c. on any ſum of money when it is at (or under) one per cent.

Rule. Divide the given ſum by 100, and the quotient is the commiſſion, brokage, or inſurance, &c. at one per cent. If the rate per cent be ¼, ½, or ¾ then take the parts of the ſaid quotient according to the rule of practice.

EXAMPLES.

1ſt What is the inſurance on 480*l.* at 1*l.* per cent per annum?

Stated according to the rule of Three, thus.

```
       £     £      £
If    100 :  1  ::  480
                      1
                  ------
                  4|80
                   |20
                   |--
             s. 16|00
```

THIS you ſee clearly ſhews you the reaſon of the above rule, at 1 per cent, conſequently if it had been $\frac{1}{2}$ $\frac{3}{4}$ or $\frac{1}{4}$ per cent you muſt take ſuch parts of 4*l*. 16*s*

2d What may a broker demand for brokage, when he ſells goods to the value of 500*l*. 10*s*. 7*d*. and I allow him 7*s* per cent?

Anſwer 1*l*. 15*s*. 0*d*. $\frac{1}{4}$

3d Suppoſe I employ a broker, who ſells goods to the value of 909*l*. 14*s*. 10*d*. what is the brokage at 6*s*. 6*d*. per cent.

Anſwer 2*l*. 19*s*. 1*d*. $\frac{3}{4}$

SECTION 4th.

TO find the intereſt of any ſum, when the rate per cent, is $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, more than the pounds given in the ſaid rate?

Rule. Multiply the principal by the pounds, in the rate per cent, as before; and let the parts for $\frac{1}{4}$, $\frac{1}{2}$, or $\frac{3}{4}$, be taken from the principal, and added to the product; then proceed according to ſection 1ſt, or 2d.

EXAMPLES.

1ſt My factor ſends me word, that he has bought goods to the value of 500*l*. 13*s*. 6*d*. upon my account; I demand what his commiſſion comes to at $3\frac{1}{2}$ per cent?

Anſwer 17*l* 10*s*. 5*d*. $\frac{1}{2}$

2d A merchant in London wants to draw upon his correſpondent in Edenburgh, and is willing to allow $1\frac{1}{8}$ per cent, exchange; what ſum muſt his bill be drawn for, that he may receive 500*l*. neat?

Anſwer 505*l*. 8*s*. 6*d*.

3d What is the amount of 400*l*. for 2 years, at $5\frac{1}{4}$ per cent, per annum. *Anſwer* 442*l*.

```
  £
 400          400
   5           42
 ----         ----
 2000        £442 amount
  100
 -----
 21|00
  2
 -----
 42£
```

S. I do not underſtand why you take $\frac{1}{4}$ of the principal for the $\frac{1}{4}$ per cent.

M. For the very ſame reaſon, that dividing the principal by 100 gives you the intereſt at 1 per cent: for you muſt note, that after you have taken $\frac{1}{4}$ of the principal, that it is afterwards divided by 100, which is the ſame, as if you had divided the principal 1ſt by 100, and then taken $\frac{1}{4}$ of that quotient.

SECTION 5th.

WHEN it is required to find the intereſt of any ſum of money, for any certain number of weeks,

Rule. Firſt find the intereſt of the given ſum for a year: then ſay

As 52 weeks (which are the weeks in a year) are to the intereſt ſo found; ſo are the weeks given, to the intereſt required.

1ſt What is the intereſt of 650*l.* for a week; at 5 per cent, per annum?

Anſwer 12*s.* 6*d.*

2d What is the intereſt of 126*l.* 12*s.* for 16 weeks, at 4½ per cent per annum.

Anſwer 1*l.* 15*s.* 0*d.* ½

3d What is the amount of 500*l.* for 20 weeks, at 3½ per cent per annum?

Anſwer 506*l.* 14*s.* 7*d*¼

SECTION 6th.

TO find the principal, when the amount, time, and rate per cent, are given.

Rule. As the amount of 1 hundred pounds, at the rate and time given,

Is to 100*l.*
So is the amount given,
To the principal required.

1ſt What principal being put to Intereſt, for 3 years, will amount to 515*l.* 4*s.* at 4*l.* per cent per annum.

Anſwer 460*l.*

2d What principal being placed out at intereſt, for 10 years, at 3½ per cent per annum, will amount to 739*l.* 16*s.*

Anſwer 548*l.*

3d What principal being put to intereſt, will amount to 876*l.* 8*s.* in 8 years, at 5 per cent per annum?

Anſwer 626*l.*

ALL the above queſtions you ſee are no more than ſimple operations in the Rule of Three; for having found out the intereſt of 100*l.* at the time and rate given, which (in the 1ſt example) is 12*l.* I add that intereſt to its principal, viz. 100*l.* which gives me the amount. Then by a ſtating in the Rule of Three Direct I ſay, as 112, which is the principal and intereſt of 100*l.*

is to the principal alone; ſo will the given amount (which is alſo principal and intereſt) be to its principal; which being wrought according to the Rule of Three Direct, gives 460 for the anſwer.

SECTION 7th.

TO find the rate per cent, when the amount, time, and principal are given,

Rule. Firſt from the amount, ſubtract the principal, and the difference will be the intereſt of the ſaid principal, for the given time: then ſay

As the whole principal,
Is to the intereſt for the whole time;
So is 100*l* to its intereſt for the ſame time.

DIVIDE the intereſt ſo found, by the time, and the quotient will be the rate per cent.

1ſt At what rate of intereſt per cent, will 460*l.* amount to 515*l.* 4*s.* in 3 years time?
Anſwer 4 *per cent.*

2d At what rate of Intereſt per cent, will 548*l.* amount to 739*l.* 16*s.* in 10 years time?
Anſwer 3½ *per cent.*

3d At what rate of intereſt per cent, will 626*l.* amount to 876*l.* 8*s.* in 8 years?
Anſwer 5 *per cent,*

THE reaſon of this rule is evident, and therefore needs no further definition.

SECTION 8th.

TO find the time, when principal, amount and rate per cent, are given.

Rule. Subtract the principle from the amount, and the difference will be the intereſt; for the time required; then ſay,

As the intereſt of the principal for 1 year, (at the given rate,
Is to 1 year;
So is the whole intereſt,
To the time required.

EXAMPLES.

1ſt In what time will 460*l.* principal, amount to 515*l.* 4*s.* at 4 per cent per annum?
Anſwer 3 *years.*

2d In what time will 548*l.* amount to 739*l.* 16*s.* at 3½ per cent per annum?
Anſwer 10 *years.*

3d In what time will 626*l.* amount to 876*l.* 8*s.* at 5*l.* per cent per annum?
Anſwer 8 *years.*

Note. The examples in the 3 laſt ſections, prove each other.

SECTION 9th.

TO find the intereſt of any ſum for any number of days.

Rule. State and work your queſtion as directed in the Rule of Five.

EXAMPLES.

WHAT is the intereſt of 126*l*, for 75 days, at 4 per cent per annum?

	£		*days*		£.*Int*		£		*days*
Stated thus If	100	:	365	:	4	::	126	:	75

which being wrought according to the rule, in caſe 1ſt, in the Rule of Five, will give the anſwer to the queſtion.

Note. When the rate is 5 per cent, and the ſum whoſe intereſt is required, conſiſts entirely of Integers;

THEN multiply the number of pounds by the ſaid rate per cent, and divide the product by 100, and the quotient will be the anſwer in ſhillings; which being again divided by 20, will give you pounds. As for inſtance, if the intereſt of 240*l.* for 70 days were required, at 5 per cent; It would juſt be this queſtion in the Rule of Three.

	days		*£*		*days*
If	365	:	240	: :	70

S. I thought in all queſtions in the Rule of Three, after you had multiplied your 2d and 3d terms together, and divided by the 1ſt, that the quotient would always be of the ſame name with the middle term; how then does it happen that the quotient ſhould be ſhillings here?

M. Becauſe in all queſtions when the rate is 5 per cent per annum, the intereſt of the ſum (whatever it is) for a year, or 365 days, will be juſt ſo many ſhillings: ſo in the foregoing queſtion, the intereſt of 240*l.* for a year at 5*l.* per cent, will juſt be 240 ſhillings, and conſequently the anſwer muſt be ſhillings.

BUT as the intereſt for days is more conveniently performed by Decimals, I ſhall forbear giving any more examples in this place.

Of Compound Interest.

COMPOUND Intereſt is that which ariſes from any principal and its Intereſt together, as the intereſt ſtill becomes due; and is called *intereſt upon intereſt.*

S. I thought the Laws of England had not allowed of Compound Intereſt, nor any premium above 5 per cent, to be taken for the loan of money.

M. On money lent upon bond, &c. The law neither allows *Compound Intereſt*, nor any rate above 5 per cent to be taken; which Law, I ſuppoſe, is founded on the following reaſons, viz. That a perſon (either through neglect, or being ignorant of the manner in which it ſo ſpeedily accumulates, to a large ſum) might be ruined before he is aware of it: and therefore ſhould the lender ſuffer the Intereſt to reſt in the borrower's hands, for any number of years whatever, he could oblige him to pay no more than the yearly Intereſt of the firſt PRINCIPAL, or ſum lent him. Yet as the lender may demand his intereſt yearly; he may therefore lend it back to the ſame perſon upon freſh ſecurity, which is in fact *Compound Intereſt*, only in this caſe, the law preſumes that the borrower cannot be ignorant of the conditions, for which he has granted a freſh inſtrument. As therefore in this caſe it becomes lawful, ſo conſequently in purchaſing annuities, penſions, or leaſes in reverſion; the buyer is allow'd compound intereſt for ready money; and therefore very neceſſary to be underſtood. But as this is alſo performed (not only) much ſhorter, but more exactly ly by decimals; I ſhall therefore give you but a few examples in this place.

When therefore it is required to find the compound intereſt of any ſum for any term of years.

Firſt, find the amount of the given ſum by ſimple intereſt, for the 1ſt year, which will be the principal for the 2d year; then find the amount of that principal for the 2d year, and that is the principle for the 3d year; and ſo on for any number of years given.

2d. Subtract the principal given from the laſt amount, and the remainder is the compound intereſt required.

EXAMPLES

1ſt What will 720 pounds amount to, in 4 years at 5 per cent per annum, compound intereſt?

Anſwer 875*l.* 3*s.* 3*d.*

```
                 720                      720
                   5                       36
Intereſt       -----                    ------ or, principal
for 1 year,   36,00              amount 756 for 2d year.
                                          5
                                        -----
                                        37,80
                                           20
                                        -----
                                        16,00
                           756
                            37 - 16
                           --------
               Principal 793 - 16 for the 3d year.
```

Principal 793 - 16 for the 3d year,

5

39,69 - 0

20

13,80

12

9,60

4

2,40

	£	s	d		£	s	d
	793	16			833	9	9½
	39	13	9½		41	13	5¾
Principal for the 4th year.	833	9	9½		£875	3	3¼ Anſ.
			5				
	41,67	8	11½				

20

13,48

12

5,87

4

3,50

2d. What is the compound intereſt of 400*l.* 10*s.* at 3½ per cent per annum?

Anſwer 43*l.* 10*s.* 8*d* ¾

3d. What is the amount of 600*l.* forborne 4 years, at 3 per cent per annum?

Anſwer 675*l.* 6*s.* 1*d* ¼

4th. What is the amount of 523*l.* 10*s.* forborne for 9 years, at 6 per cent per annum compound intereſt?
Anſwer 863*l.* 8*s.* 10*d* $\frac{1}{4}$

5th What will 480*l.* amount to in 6 years, at 5 per cent per annum compound intereſt?
Anſwer 643*l.* 4*s.* 10*d* $\frac{1}{4}$

CHAP. VII.

Of REBATE or DISCOUNT.

REBATE or Diſcount is, when a certain ſum of money, due at any time to come, is ſatisfied by paying ſo much preſent money, as being put out to intereſt, would amount to ~~the given ſum~~ in the ſame time. To find therefore the diſcount of any ſum, according to the time it would have become due, this is the

Rule. 1ſt Find the intereſt for 100*l.* at the rate per cent and for the time given.

2d Add the intereſt ſo found to the 100*l.* and ſay,

3d As that ſum (which is principal and intereſt together) Is to the intereſt alone,

So is the given ſum, to the rebate (or its intereſt)

4th Subtract the rebate from the whole amount, and the remainder is the preſent worth. Or thus,

3d As the ſaid principal and intereſt, are to 100*l.* (the principal alone)

So is the given ſum to its principal (which is the preſent worth)

4th Subtract the preſent worth from the given ſum, and the remainder is the rebate.

TO prove queſtions in this rule. Find the amount of the preſent payment, at the time and rate per cent given, which will be equal to the 1ſt given ſum.

EXAMPLES.

1ſt What is the diſcount of 500*l.* for 10 months, at 5 per cent per annum?

1ſt I find the intereſt of 100 pound for the ſaid term, in this manner. } ſaying If 12 : 5 :: 10

5

12) 50 (4*l*
48

2
20

12) 40 (3*s.*
36

4
12

12) 48 (4*d*
48

Then I add the intereſt ſo found to 100*l* }

£	*s*	*d*
100	0	0
4	3	4
104	3	4

* * *

And laſtly I ſay

```
     £  s d    £ s d      £
As 104  3 4  : 4 3 4  ::  500
      20        20         20
   -------   -------   --------
    2083       83       10000
      12       12          12
   -------   -------   --------
   25|000     1000     120000
                         1000
                      --------          12
           25|000) 120000|000 ( 4800
                    100           ------
                   -----          2|0)40|0
                     200          ------
                     200           £20
                   -----
                      00
```

S. I do not clearly comprehend the difference between the Diſcount and Intereſt of any particular ſum, for inſtance, if I had any perſons note, or accepted bill for 100*l.* payable 12 months after date, and had an immediate occaſion for the caſh, would not the abatement which I ought to make them for preſent payment, be exactly 5*l.*

M No! for in that caſe you would be a looſer; for you are to conſider that had you waited till the bill had become due, you would then have had 100*l.* to receive; therefore you ought to have ſuch a ſum in preſent money, as (if put out to intereſt for the ſaid time; at the ſaid rate per cent,) would have again amounted to 100*l.* which 95*l.* would not have done.

AGAIN conſider in the next place; that if the perſon whoſe acceptance you had, had kept the money in his hand till the money became due; he could only (at 5 per cent) made 5*l.* of it, at the end of 12 months; whereas by your paying him 5*l.* immediately in hand; he could lend that 5*l.* out, which in 12 months would gain him 5 ſhillings more; ſo that inſtead of 5*l.* he would have 5*l.* 5*s.* The true diſcount therefore which you ought to have allowed ſhould have been 4*l.* 15*s.* 2*d*¾ and ſome fractions over.

BUT had the bill been for 105*l.* then 5*l.* would have been exactly the diſcount; for had he kept the money, till the expiration of the 12 months, he would then have made 5*l.* 5*s.* intereſt; and by you allowing him 5*l.* for preſent payment; that 5*l.* being put out at intereſt, would likewiſe amount to 5*l.* 5*s.* which you ſee is right, with reſpect to *him.* Let us next examine if it be right with reſpect to *yourſelf*: and 1ſt then, conſider that if you had waited till the bill had become due, viz, 12 months, you would then have had 105*l.* to receive. Now if you had 100 pounds paid immediately, and, put it out to intereſt at the ſame rate per cent, viz. 5, it is plain at the 12 months end, you would then have alſo 105*l.* to receive. If therefore it is the ſame with reſpect to each, it conſequently muſt be equal, or juſt, between both.

2d I demand the preſent worth of 473*l.* 5*s.* 6*d.* payable 7 months hence, diſcount allowed thereon, at 6½ per cent per annum?

Anſwer 455*l.* 19*s.* 8*d.* ½

3d What is the rebate for 975*l.* 11*s.* 2*d.* for 11 months, at 6*l.* per cent?

Anſwer 42*l.* 9*s.* 5*d.* ½ [illegible] over

4th What preſent money will ſatisfy a debt of 85*l* 10*s.* due at the expiration of 10 months, diſcount allowed at the rate of 6*l.* per cent per annum?

Anſwer 81*l.* 8*s.* 7*d.*

5th Suppoſe 810*l.* were to be paid 3 months hence, allowing 5 per cent diſcount, what muſt be paid in hand?

Anſwer 800*l.*

6th A young gentleman has a legacy of 400*l.* left him by his uncle, due the 4th of November, but wanting money, is willing to allow 4½ per cent diſcount, for preſent payment; what muſt he receive, this being the 9th of April?

Anſwer 390*l.*

7th Sold goods upon truſt for 8 months, to the amount of 95*l.* 10*s.* what is the preſent worth at 7 per cent?

Anſwer 91*l.* 4*s.* 10*d.*

8th How much ready money, for a note of 18*l* due 15 months hence, at 5 per cent?

Anſwer 16*l* 18*s* 10*d.*

9th What is the preſent worth of 520*l.* due 6 months hence, at 5 per cent?

Anſwer 507*l.* 6*s.* 4*d.* ¼

10th What is the diſcount of 300*l.* for 3 years, at 5 per cent per annum, Simple Intereſt?

S. The operation in theſe queſtions are ſo very eaſy, and the reaſons of it you have made ſo very clear; that I think you need give no further demonſtration of it.

M.

M. You have been miſtaken before, and perhaps may be again, tell me therefore how you would perform the laſt queſtion.

S. I would find the diſcount of 300*l.* for 1 year, according to the foregoing rule; and then I would multiply that diſcount by 3, which would give me the diſcount for 3 years.

M. You here again verify my obſervation, in page 91 and 92, viz. how eaſily a perſon (in attempting to contract his work) may fall into an error, for want of being thoroughly acquainted with the *principles* on which the rule is founded. This method I own has ſome ſhadow of reaſon in it, but why it ſhould not Produce the ſame effect as the true method of working it, will beſt appear by giving both operations, at large.

See both Operations, in the next Page.

Wrong way,
£ £ £
As 105 : 5 :: 300
5
105)1500(14
105
450
420
30
20
105)600(5
525
75
12
105)900(8
840
60
4
105)240(2
210
30

£ s d
14 : 5 : 8 $\frac{1}{2}$
3
£42 : 17 : 1 $\frac{1}{2}$ wrong answer.

Right way,
£ £ £
As 115 : 15 :: 300
15
1500
300
115)4500(39
345
1050
1035
15
20
115)300(2
230
70
12
115)840(7
805
35
4
115)140(1
115
25

[*NOTE.*]

HERE you ſee the *Two Operations* produce *different anſwers*; and altho', at firſt ſight, it may ſeem ſtrange why they ſhould; yet upon a more particular enquiry, the reaſons will appear evident. Firſt then, Let us view the two different manners in which this queſtion is ſtated; there we find, according to the RIGHT method, that after having added the Intereſt of 100 pounds for 3 years, at 5 per cent (which is 15*l*) to 100*l*; the ſum is 115*l*; which is the *Firſt Term* in your ſtating: the 15*l*, which is the Intereſt itſelf, is the *Middle Term*; and the ſum whoſe diſcount is required, is the *Laſt*. Again, according to the WRONG method of ſtating it, 100*l*, with its intereſt for one year, added to it, (which is 105*l*) is the *Firſt Term*; the Intereſt itſelf, viz. 5*l*, is the *Middle Term*; and the *Laſt*, the ſame as before. The reaſon then why multiplying the Diſcount of 300 pound for *One* year, by *Three*, cannot be equal to the Diſcount of 300 pounds for *Three* years, will appear clear from conſidering well—

The Firſt *and* Second Terms *in each Stating.*

Now here you ſee that the 1ſt and 2d terms, by the RIGHT WAY, are 115 and 15; and the WRONG WAY, 105 and 5. But, both operations being performed according to the RULE of THREE DIRECT, you will perhaps ſay, *That altho' the* Firſt *and* Second *terms in each ſtating differ; yet they may differ in ſuch proportion, one to the other, as may produce the ſame effect.* This I grant: but it being equally true, "That when 'any TWO terms, in ONE ſtating, agree with the 'ſame TWO terms in ANOTHER ſtating, the other 'term, in each, muſt of neceſſity be the ſame, to pro'duce the ſame anſwer.' But two of the terms, in each of thoſe ſtatings, are the ſame; and the remaining term in each, differ one from the other: conſequently, cannot produce the ſame anſwer.

Scho. Pray, how do you prove any two terms in the one ſtating, equal to the ſame two terms in the other?

Maſt. As they ſtand ranged in the ſtatings indeed, the laſt term of each (only) appears to be the ſame; but in the courſe of the operation, the middle terms of each are likewiſe made to agree; for whether (according to the wrong method) you multiply your laſt term by 5, and divide that product by 105, and then multiply that quotient, again by 3; the effect is the ſame, as if you had multiply'd your laſt term by 15 at once, and divided by 105. Hence it is plain, that the middle term is likewiſe 15, or made equal to it by the operation, whereas the firſt term is 105. As therefore the two terms which you multiply into each other are the ſame in both; and the firſt term in each, by which you divide, differ one from another; the anſwers conſequently cannot agree; for the diviſor in the one is 105, and in the other 115.

CHAP. VIII.

Of EQUATION of PAYMENTS, The COMMON WAY.

EQUATION of *Payments* is, when ſeveral ſums of money, due at different times, are reduced to one *mean time* for the payment of the whole, without loſs to debtor or creditor; to perform which this is the

Rule. Multiply each payment by its time, and divide the ſum of all the products by the whole debt, the quotient is the equated time.

EXAMPLES.

1ſt. A owes B 200*l.* whereof 100*l.* is to be paid at 6 months, and the other 100*l.* at 12 months, but they agree to reduce them to one payment, when muſt the whole be paid?

Anſwer 9 months.

2d A owes B goods to the amount of 400*l.* of which 100*l.* was to be paid at 3 months; 100*l.* at 7 months; and the reſt at 9 months; but they agree to reduce them to 1 payment: when muſt the whole be paid?

Anſwer 7 months.

3d C owes D 1000*l.* whereof 120*l.* is to be paid immediately, 400*l.* at 5 months, and the reſt at 10 months, but they agree to make 1 payment of the whole; I demand the equated time?

Anſwer 6 months.

5th E owes F a certain ſum of money, which is to be paid at 4 ſeveral payments, viz. $\frac{1}{4}$, at 1 month, $\frac{1}{4}$ at 2 months, $\frac{1}{4}$ at 3 months, and $\frac{1}{4}$ at 6 months, but they agree to make 1 payment of the whole, I demand when that time muſt be?

Anſwer 3 months.

5th G ſtands indebted to H a certain ſum of money, which is to be paid $\frac{1}{2}$ preſent, $\frac{1}{4}$ at 4 months, and the reſt at 8 months; which is the equated time?

Anſwer 3 months.

6th K owes L 120*l.* which will be due 6 months hence, but K is willing to pay him 60*l.* in hand, provided he can have the reſt forborn a longer time, it is agreed on; the time of forbearance therefore is required.

Anſwer 7 months.

THE reaſon of this rule is founded on the following principles, namely, *That the intereſts of the ſeveral ſums, from the time they become due, to the time of the laſt payment, being added together; ſhall be equal to the Intereſt of the whole ſum, from the equated time, till the ſaid time of the laſt payment.*

S. This indeed ſeems very equitable; but I cannot ſee why the operation (according to the given rule) ſhould produce the effect.

M. For this reaſon, firſt by multiplying each particular payment, by the reſpective time it becomes due, (ſuppoſing 1 of them 50*l*. at 3 months) the product will be a ſum, which if lent out on intereſt for a month, at any certain rate per cent per annum, the intereſt will be equal to the intereſt of 50*l*. for 3 months, at the ſame rate per cent; and ſo of all the reſt.

THE ſeveral products then, of the particular payments, multiplied by the reſpective time at which they become due, being added together; the proportion will then be thus. *If the total ſum of thoſe products in* 1 *month, give a certain intereſt; in what time will the whole debt gain the ſame intereſt?* Now as the ſum of the ſeveral payments added together (which is the whole debt) muſt be leſs than the ſum of the product of each payment multiplied by its reſpective times; the ſtating will therefore always be Inverſe. Again, the *middle term* being always 1 year, 1 month, 1 week, &c. conſequently the *firſt term* which ought to be multiplied by it, remains the ſame; which being divided by the *laſt term*, gives the equated time. Nothing further, I think, need be ſaid, to prove the general rule; provided you have carefully noted, of what the *firſt* and *laſt* terms of your ſtating are compoſed; which you may eaſily do by reading the laſt page over again.

Scho. But there is ſtill one part of this operation which ſeems to contradict the reaſon on which, as you told me, this rule was founded; which is this, *Inſtead of finding the intereſt of every particular payment from the time it becomes due, till the time of the laſt payment; It gives the intereſt of each payment from the preſent time, till the time it becomes due*: which muſt differ more or leſs from truth, as ſuch times of payment are nearer to, or further from the *mean time*, between the preſent time, and the time of the laſt payment. As for inſtance; *if the laſt payment were to be at* 9 *months, and the firſt payment at the end of* 3. Here I ſhould think, that inſtead of multiplying ſuch payment by 3, that it ſhould rather be multiplyed by 6, this being the time between that, on which it became due, and the laſt payment; which is the true time which ſuch ſum could bear intereſt; and conſequently when we multiply it by *Three*, we muſt loſe three months intereſt. Again, *if the 2d payment was at* 7 *months, inſtead of multiplying* the ſum due at that time by 7, I think it ought to be multiplied by 2, as it could only bear intereſt 2 months; and ſo of all the other payments.

M. Your obſervation is very judicious; but I ſhall prove to you, that the other way anſwers the ſame purpoſe, and is leſs trouble. For if we were to proceed according to your method, and multiply every payment by the diſtance of time between that, on which they become due, and the laſt payment; (leaving the ſaid laſt payment out, as it could bear no intereſt) and divide the ſum of all their products by the whole debt; it would give us the time in which the whole debt would amount to a certain intereſt: which time being ſubtracted from the time of the laſt payment, would give the equated time at which the whole debt ought to be paid: which ſum would alſo for the remaining time amount to the ſame intereſt. Now if we ſuppoſe the laſt payment to be 9 months, and the equated time at which the whole ought to be paid, 6; The intereſt therefore of the whole ſum for 3 months, being equal to the intereſt of the ſeveral payments from the time they become due, to the time of the laſt payment; conſequently the intereſt of the ſeveral ſums from the preſent time, to the time on which they become due, muſt be equal to the intereſt of the whole ſum for 6 months, the equated time: which was to be proved.

S. Why then is this called EQUATION of PAYMENTS the *common way?* for if you don't mean ſome ſhorter method of finding the ſame anſwer, I cannot ſee how a different one can be more equitable.

M. There is ſtill a more exact way of performing it, which I hope I ſhall likewiſe be able to prove to you: for equity alſo requires, that the preſent worth (which you may find by diſcount) of the whole debt, from the equated time, ought to be equal to the preſent worth of all the particular payments, from the time which each of them became due.

S. This ſeems a neceſſary conſequence of the other, for I cannot conceive that any other time can be aſſigned for the payment of the whole without deſtroying thoſe reaſons which are the foundation of this rule; and which were ſo full, clear, and ſelf evident; that I think it impoſſible to ſubſtitute any other in their place, which will carry equal, much leſs ſuperior force with them.

[*The Reader is deſired to take particular Notice of this, and the following Page.*]

* * *

M. I ſhall only add other reaſons, without weakening thoſe already given: let us therefore again ſuppoſe, that a perſon had 300*l.* due to him, to be paid to him at 3, or any other number of ſeveral payments; and the debtor was willing to agree upon a time, for the payment of the whole; which we ſhall alſo ſuppoſe (according to the rule given) to be 6 months, and for which the creditor takes the debtor's note, payable at the ſaid time. The creditor having an immediate demand for caſh, applies to a perſon to diſcount this note for him, which he does at a certain rate per cent. Now if the aforeſaid method of finding the equated time were true; this ſum which he received, being put out at a certain rate of intereſt, ought to amount to the ſame ſum at the expiration of the laſt payment, as the ſeveral ſums would have done, from the time that each of them became due, to the time of the ſaid laſt payment, at the ſame rate of intereſt.

Scho. I can ſee no reaſon why it ſhould not.

Maſt. The reaſon then is this. Firſt, it is plain, that the ſum which he received for the note, being put out at the ſame rate of intereſt, and for the ſame time at which it was diſcounted, would again amount to 300*l* principal and intereſt together; which ſum indeed bearing intereſt from that time, to the laſt payment, would be equal to the amount of the ſeveral payments together, with intereſt from the time on which they reſpectively became due, to the time of the laſt payment. But you are to conſider that we are only valuing it at ſimple intereſt, and therefore the intereſt of the ſum which he received for the note, ought not to have been added to the principal at the end of the ſaid 6 months. It is therefore evident, that had the neat ſum which he received for the note, continued to the time of the laſt payment (without adding its intereſt to it at the end of 6 months) which according to ſimple intereſt it ſhould have done, that it would not have amounted to the true ſum which it ought to have done; and conſequently the creditor muſt have been a looſer by the equated time found according to the aforegoing rule.

Scho. I ſhall be obliged to you then, if you will ſhew me the true method of finding the equated time.

Maſt. This cannot exactly be performed without Decimals, on account of the remainders which may happen, however I ſhall ſhew you the method in which it is done. Firſt, according to the rule of *Diſcount* (in the laſt chapter) find out the preſent worth of every particular payment, acording to the time it becomes due, by rebating at the rate of intereſt agreed on: Then find in what time the total ſum of all thoſe preſent worths, will amount to the whole debt (according to the rule in ſection 8th on intereſt, page 152) at the ſame rate per cent, and the anſwer ſo found, ſhall be the equated time for the payment of the whole.

* * *

S. Will the amount of the whole debt, from the true equated time, to the time of the laſt payment; be equal to the amount of the ſeveral payments from the time they become due to the ſaid laſt payment; the ſame as by the common way of finding the equated time.

M. Yes for it could not otherwiſe have been true.

S. I ſhall only trouble you with one queſtion more, and then I have done; and that is, how the equated time is to be found, when neither the whole debt, nor the particular payments which make up the whole debt, are mentioned in the queſtion; which (according to the rule) ought to be multiplied by their reſpective times on which they become due?

M. You may ſuppoſe any ſum at pleaſure, which will divide equally into ſuch parts as are named in the queſtion; which will anſwer all ſums whatever: for as the parts of any one ſum, are to the whole; ſo will the parts of any other ſum be, to their whole.

Note. When a ſum of money is due, and the time at which all or any of the ſeveral payments are due, exceed that of a year; then the equated time is found out, by firſt finding the preſent worth of each payment, by diſcounting at compound Intereſt; and then finding again in what time the total ſums of each preſent worth, will amount to the whole debt, at the ſame rate of Compound Intereſt. But of this I ſhall treat more fully, when I come to ſhew the conſtruction of Decimal Tables, for finding the preſent worth of annuities.

CHAP. IX.

Of BARTER.

BARTER is the exchanging of 1 commodity for another, and teaches merchants ſo to proportion their quantities (according to the value of the goods) as neither may ſuſtain loſs.

AS this rule is alſo a branch of the Rule of Three, it would be needleſs to repeat the rule over again, by which the operation is performed, and therefore ſhall proceed to give you ſome

EXAMPLES.

1ſt. How much ſugar at 9*d.* per *lb* muſt be given in Barter for 8*cwt.* of tobacco at 14*d.* per *lb.*

Anſwer 12*C* 1*qr.* 21*lb.* 12*oz.* 7*drs.*

2d A and B bartered: A had 5 *cwt.* of ſugar, at 6*d.* per *lb:* which he gave for a quantity of cinnamon, at 10*s.* 8*d.* per *lb.* I demand how much cinnamon B gave A for his 5*cwt.* of ſugar?

Anſwer 26*lb.* 4*oz.*

3d. B gave to C, 3 *hhds.* of brandy at 6*s.* 8*d.* per gallon for 126 yards of cloth, I demand what the cloth ſtood B in per yard?

Anſwer 10*s.* *per yard.*

4th A hath linnen cloth worth 2*s.* 6*d.* per yard, ready money, but in barter he will have 3*s.* per yard for it: B has broad cloth at 10*s.* per yard, ready money: I demand how much broad cloth B ought to give A for 500 yards of his linnen, and at what rate he ought to rate his broad cloth per yard, to be upon an equal footing with A.

Anſwer 125 *yards of broad cloth at* 12*s.* *per yard.*

I ſhall here give you the operation of the laſt example in full, which will I hope ſufficiently inſtruct you how to perform any queſtion in Barter.

s d s s
1ſt. If 2 6 : 3 :: 10
2 2
— —
5 20
3
—
5)60
—
12*s.*

yds. s yds.
2d. If 1 : 3 :: 500
3
—
1500

s yds. s
3d. If 12 : 1 :: 1500
1
—
12)1500
—
125 *yds.*

Here I firſt find what B ought to value his cloth at per yard, in proportion to A's advance upon his linnen. Saying if 2*s.* 6*d.* advance to 3*s*; what ought 10*s.* advance to; which by the operation I find to be 12 ſhillings. I next ſee what A's 500 yards of linnen will come to at 3*s.* per yard, which I find to be 1500*s.* And laſtly I ſay, If 12*s.* buy 1 *yard*, how many yards will 1500 ſhillings buy? which gives 125; ſo that I conclude B ought to give A 125 yards of his broad cloth at 12*s.* per yard, for 500 yards of linnen at 3*s.* per yard.

CHAP. X.

Of LOSS and GAIN.

BY this Rule we not only diſcover the whole Gain or Loſs upon any commodity, (having the price they coſt, and the price they ſold for both given) but alſo at what rate we gain or looſe per cent; and altho' there is nothing new in it, beſides the title, (as it is founded on the ſame principals with the *Rule of Three, Intereſt, Diſcount*, &c.) yet it was neceſſary to treat of it in a chapter by itſelf, in order that I might under one head, from thoſe three ſeveral rules, form ſome general ones, for the more particular information of the learner, in a point ſo *very eſſential to all merchants, traders*, &c. and which I ſhall give in the following caſes.

Caſe 1ſt. When the price is given, what 1 *yard pound, cwt*, &c. ſold for; to know what you gain'd, or loſt upon the whole; This is the

Rule. Firſt find (according to the *Rule of Three direct*) what the whole number of *yards. pounds, cwt*. &c. comes to, and alſo what the whole ſold for; and ſubtracting the one from the other, the *difference* will be the *whole gain* or *loſs*.

Caſe 2d. If it be required to find the *gain* or *loſs* per *cent*

Rule. Firſt find the whole *gain* or *loſs*, according to the rule in caſe 1ſt.

Then ſay, as the whole coſt, is to the whole gain or loſs; So is 100*l*. to its gain or loſs: which will give the anſwer.

Caſe 3d. When the rate per cent is given at which a perſon gain'd when he ſold the *yd. lb. cwt*. &c. for a certain price; and the prime coſt of one *yd. lb. cwt*, &c. is required.

Rule. Firſt add the rate per cent to 100*l*; Then ſay,
As that ſum, is to 100*l*.
So is, the price which 1 *yd. lb. cwt.* &c. Sold for,
To the prime coſt.

Caſe 4th When the rate per cent is given at which a perſon gains, when he ſells the *yd. lb. cwt.* &c. for a certain price; and it is required to know; *at what rate per cent he would gain by ſelling the yd. lb. cwt, &c. at any other certain price propoſed.*

Rule. Add the rate per cent at which he gain'd (by ſelling it at the firſt price) to 100*l*: Then ſay,
As the *firſt price*, for which he ſold the *yd. lb. cwt*, &c.
Is to the *ſaid ſum*;
So is the *other price* at which it is propoſed he ſhould ſell the *yd. lb. cwt.* &c.
To *a 4th number.* Laſtly

From the 4th number ſo found, ſubtract 100*l*; and the remainder will be the rate per cent at which he would gain, by ſelling at the price propoſed.

EXAMPLES.

UNDER CASE 1ſt.

1ſt. Bought 872 yards of cloth at 8*s*. 6*d*. per yard, and ſold the ſame at 10*s*. 4*d*. per yard, what is gain'd by the whole?

Anſwer 79*l*. 18*s*, 8*d*.

2d If I buy deals at 2*s*. a piece, and ſell them again at 20*d*. what will I loſe by 132 dozen?

Anſwer 2*l*. 4*s*.

EXAMPLES.

UNDER CASE 2d.

1ſt Bought 19 fodder of lead, each 19 *cwt.* 2 *qrs*.

at 14*s.* per *cwt.* whether do I gain or loſe when I ſell it at 4*d.* per *lb.* and how much per cent?

Anſwer he gained 432*l.* 5*s.* on the whole, and at the rate of 166*l.* 13*s.* 4*d* per cent.

2 A grocer bought 6*cwt.* 3*qrs*, of cloves, at 2*s.* 4*d.* per *lb.* and ſold the ſame again for 105*l.* 8*s.* what did he gain on the whole, and at what rate per cent?

Anſwer 17*l.* 4*s.* on the whole, and 19*l.* 4*s.* 3*d.* per cent.

EXAMPLES.

UNDER CASE 3d.

1ſt If when I ſell my cloth at 5*s.* per yard, I gain 8*l.* per cent; I demand what a yard coſt me at that rate?

Anſwer 4*s.* 7*d*½ and ſome-what over.

2d If when I ſell a *hhd.* of wine for 20*l.* I gain 15*l.* per cent; I demand what the *hhd.* coſt me?

Anſwer 17*l.* 7*s.* 9*d*¼

EXAMPLES.

UNDER CASE 4th.

1ſt If when I ſell my cloth at 5*s.* per yard, I gain 8*l.* per cent; what do I gain per cent when I ſell it at 6*s.* 3*d.* per yard?

Anſwer 35*l. per cent.*

2 If when I ſell deals at 15*d.* a piece, I gain 2*l.* per cent; what do I gain when I ſell them at 1*s.* 6*d.* a piece?

Anſwer 30*l.* 16*s. per cent.*

Note. Although there may be many queſtions propoſed in Loſs and Gain, which do not immediately fall under any of the foregoing caſes; yet theſe being well underſtood, the learner will find no great difficulty in ſolving any queſtion which can be propoſed in this rule. That I may therefore be able to judge how far you are maſter of theſe 4 caſes, let me ſee if you can work the 1ſt example in the laſt caſe.

S. That would be giving you but a very weak proof of my abilities; ſince you have given ſo plain a rule for it, that it is next to an impoſſibility to err; and therefore I ſhall give you a better teſtimony of my ſkill, by performing the ſame queſtion by a method much ſhorter, and leſs complex, which is this; If 5*s.* gain 8*l.* per cent; what will 6*s.* 3*d.* gain per cent?

M. You were unwilling to give me ſo weak a proof of your abilities, as to work by the *rule given*; but you have now given me a very ſtrong one, *that you are intirely ignorant of the matter.* However go on with the operation.

S. I ſhall

s *l* *s* *d*
If 5 : 8 :: 6 3
12 12
60 75
8
6|0) 60|0
10 *Anſwer* 10 *per cent.*

Ah! it does not bring out the ſame anſwer I ſee, as by *your rule;* and yet I think there is a great deal of *reaſon* in the operation too.

M.

M. Not the leaſt reaſon in it; for you are to conſider that 5*s.* was *the price it was ſold for*; therefore it is not 5*s.* that is equal to 8 per cent, *but the difference between the prime coſt and* 5*s.* Had you therefore found (according to the 1ſt example in caſe 3d) what it coſt per yard, which you will there find to be 4*s.* 7*d* ½ and ſome fraction over; I ſay had you ſubtracted that from the 5*s.* the difference which is 4*d*½, *would have been equal to* 8 *per cent*; and then again found the difference between the *prime coſt*, and 6*s.* 3*d.* which is 1*s.* 7*d* ½, then ſaid if 4*d.*½ be equal to 8 per cent, what will 1*s.* 7*d* ½ be equal to? that would have come pretty near the truth, although not all together, as for want of knowing the uſe of fractions, you loſt the remainder. But the other method (according to the rule for working queſtions in caſe 3d) is ſtill better, as it is done with far leſs trouble.

S. I am perfectly convinced of my error, and alſo of the reaſon, why this *laſt method* you have ſhewed me, *ſhould produce the effect*: I ſhall now be obliged to you, if you will point out to me, *the reaſon* on which this rule is founded.

M. Firſt then, you are to conſider, that the 5*s.* for which the cloth ſold per yard, was *prime coſt and profit together*. Now although we do not know what the *coſt*, nor what the *profit* were ſeperately; yet we are told that the *profit* was equal to 8 per cent; wherefore conſidering 5*s.* as a *principal and intereſt added together*, and the adding likewiſe 8*l.* (which the *profit* is equal to) to 100*l.* Then the proportion is this,

As 5*s*. *principal and intereſt*,
Is to 108*l*. (which is likewiſe *principal* and *Intereſt*,
So is 6*s*. 3*d*. to a *fourth ſum*, which is alſo *principal* and *Intereſt*.

THEN ſubtracting 100*l*. which is the *principal*, from it, it leaves 35*l*. which is the *rate per cent, profit, which was gained by ſelling the cloth at 6s. 3d. per yard.*

Here follow more examples for the Exerciſe of the learner.

1ſt A broker bought goods at 4*s*. per *lb*. and ſold them again for 3*s*. 4*d*. what did he looſe per cent, at that rate?

Anſwer 16*l*. 13*s*. 4*d*.

2d Bought 40 quarters of malt at 25*s*. per quarter, and having received damage, I loſt 12 per cent by it; what did I looſe per buſhel?

Anſwer 4*d*½

If when I ſell rice at 1*l*. 14*s*. per *cwt*. I looſe 3 per cent, what do I gain or looſe per cent, when I ſell it at 1*l*. 16*s* per *cwt*?

Anſwer 2*l*. 14*s*. 1*d*½

If I give 51*l*. 15*s*. 6*d*. for 1 *cwt*. of cinnamon, at what rate ought I to ſell the ſame ſo as to gain 12*l*. per cent?

Anſwer 57*l*. 19*s*. 9*d*.

An oilman bought 7 caſks of oil at 18*l*. 10*s*. per caſk, I demand at what rate per caſk he muſt ſell the ſame, ſo as to gain 17 per cent in the whole?

Anſwer 21*l*. 12*s*. 10*d*.¾

CHAP. XI.

Of FELLOWSHIP.

THE Rules of plural proportion, are thoſe by which we reſolve queſtions that are diſcoverable by *more Rules of Three, than one*; and yet cannot be performed by any of the double rules of Three mentioned before; of theſe rules there are divers kinds and varieties, according to the nature of the queſtion propoſed: for here the terms given are ſometimes four, ſometimes five, and ſometimes more; and the terms required ſometimes more than one, &c.

The rule of *Fellowſhip* is, when *two* or *more partners* trade together with *one common ſtock*, and when they have gain'd or loſt, it ſhews how to find each man's particular gain or loſs, of which there are two kinds, *Single* and *compound*.

Of SINGLE FELLOWSHIP.

Single Fellowſhip, is when each partner's ſhare of the ſtock, continues for an equal ſpace of time.

Rule. As the ſum of all the ſtocks,
Is to the whole gain or loſs:
So is each man's particular ſtock,
To his reſpective ſhare of the gain or loſs.

Proof. Add all the particular gains or loſſes together, and the total ſum will be equal to the whole gain or loſs.

EXAM-

EXAMPLES.

1ſt. Three merchants A, B and C join ſtocks in a certain adventure: A put put in 100*l.* B puts in 150*l.* and C puts in 200*l.* they gain 90*l.* I demand each man's particular ſhare of the gain? Perform'd according to the general rule,

£
Thus, A's ſtock 100
B's ſtock 150
C's ſtock 200
——
The whole ſtock 450

Then for) £ £ £
A's ſhare ſay, If 450 : 90 :: 100
100
——
450)9000(20£
900
——
...0

£ £ £
2d. For B's ſhare, If 450 : 90 :: 150
90
——
450)13500(30£
1350
....0

3d For C's ſhare. If 450 : 90 :: 200
90
——
£
450) 18000 (40
1800
——
....0

£
Proof A's gain, 20
B's d° 30
C's d° 40
——
Whole gain 90

Note 1ſt When the whole ſtock, can be divided by the whole profit or loſs without a remainder, the work may be contracted thus. Divide the whole ſtock by the whole profit; and by the quotient, divide each partners particular ſtock, and the particular quotient will be the reſpective gain, or loſs of each man.

Reaſon. For ſo often as the whole profit is contain'd in the whole ſtock; juſt ſo often muſt each perſon's reſpective ſhare of the profit, be contain'd in his particular ſhare of the ſtock.

2d A, B and C agree to go partners for a certain time; A puts in 20*l.* B 30*l.* and C 40*l.* they gain 180*l.* I demand each man's ſhare of the gain?

Note 2d. When the whole gain or loſs can be divided by the whole ſtock without a remainder, the work may be coutracted thus,

Divide the whole profit or loſs, by the whole ſtock; and by the quotient, multiply each partner's particular ſtock, and the particular products will be the reſpective gain or loſs of each man.

Reaſon. For ſo often as the whole ſtock, is contained in the whole profit; juſt ſo often will each man's particular ſtock, be contained in his reſpective profit.

3d A, B, and C put in money together, A put in 20*l.* B and C put in together 70*l.* they gain 180*l.* of of which B took up 60*l.* what did A and C gain, and B and C put in?

Anſwer A gained 40*l.* and C 80*l.* B put in 30*l.* and C 40*l.*

HERE although we don't know B and C's particular ſhare of the ſtock, yet we are told, they together put in 60*l.* conſequently that 60*l.* together with A's ſtock, (which was 20*l*) muſt make up the whole ſtock. Having then (according to the general rule) found A's gain; we next proceed to find B's ſtock, and being told that he took up 60*l.* for his ſhare of the profit, we find out his ſtock in this manner: as 180*l.* (which is the whole profit) is to 60*l.* the profit of B; ſo will

90*l*. (which is the whole ſtock) be to his particular ſhare of the ſtock. Or as A's profit is to A's ſtock; ſo will B's profit, be to his ſtock; which ſubtracting from 70*l*. the remainder is C's ſtock; and laſtly ſubtracting the gain of A and B, from the whole gain; the remainder muſt be the gain of C.

Note 3d This and the laſt queſtion prove each other.

4th Three merchants A, B and C freight a ſhip from Madeira with wine; in which A had 240 tuns, B 144, and C 192: but meeting with a ſtorm at ſea, they were obliged to throw 48 tuns over board, I deſire to know what part of the loſs each muſt ſuſtain?

Anſwer A muſt looſe 20 tuns, B 12 tuns, and C 16 tuns.

5th A bankrupt's effects amount to 2080*l*. and he is indebted to A 1440*l*. to B 1920*l* and to C 2880*l*. how much of the effects belonged to each of them?

Anſwer A muſt have 480*l*. B 640*l*. and C 960*l*.

6th A, B, C, and D, buy a ſhip which coſt them 300*l*. of which A paid $\frac{1}{3}$, B $\frac{1}{4}$, C $\frac{1}{5}$, and D $\frac{1}{6}$: I demand the ſum which each man paid.

Anſwer A muſt pay 105*l*. 4*s*. 10*d*. $\frac{3}{4}$ ſomewhat over.
B 78*l*. 18*s* 11*d*. $\frac{1}{4}$. d°
C 63*l*. 3*s*. 1*d*. $\frac{3}{4}$ d°.
And D 57*l*. 12*s*. 7*d*. $\frac{1}{2}$ d°.

IN performing this laſt queſtion, I take $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, of 300*l*. which was the price the ſhip coſt: which ſeveral parts being added together; I find it amount to 285*l*. (for $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, are not equal to the whole of any thing, as I ſhall more fully prove when I come to treat of *Fractions*) I therefore ſay, *as the ſum of all thoſe parts*, viz. 285*l*. *is to* 100*l*. (which is the ſum they ought to be) *ſo is the ſum of each particular part, to the ſum it ought to be.*

Of DOUBLE FELLOWSHIP.

IT is called *Double Fellowſhip*, when each man's ſtock, has reſpect to the particular time which it continued in *Company:* ſo that his ſhare of the gain, muſt not only be proportioned to the ſum of money he put in; but likewiſe to the *time* it continued *in company*.

Rule. Firſt multiply each man's particular ſtock by the the time which it continued in company: Then ſay,

As the *ſum* of all thoſe products added together,
Is to the whole *gain* or *loſs;*
So is each *particular product*,
To each man's *reſpective ſhare* of the *gain* or *loſs*.

EXAMPLES.

1ſt A and B go into partnerſhip, A puts in 100*l.* for 6 months: and B put in 100*l.* for 9 months: they gained 80*l.* What is each man's ſhare of the gain?

See the operation.

	100		100		900
Stock and time of A.	6	Stock and time of B.	9		600
	600		900	total ſum	1500

£ £ £
As 1500 : 80 :: 600
80

15|00) 480|00 (32 A's ſhare.
45

30
30

	£		£
Whole gain	80	Proof	32
A's ſhare of it	32		48
B's ſhare of it	48		80

Note. As there are but 2 partners concerned here, I firſt found the gain of A; which being ſubtracted from the whole gain, the remainder is the gain of B; but if there had been more partners than 2, there muſt have been ſo many ſeveral ſtatings, as there were partner's mentioned in the queſtion.

THE reaſon why we multiply each man's ſtock by his time, I have already ſhewed you in the Double Rule of Three Inverſe: but to ſave you the trouble of turning back, I ſhall again repeat it here.

FIRST then it is plain, that 100*l.* multiplied by 6 months, muſt be equal to 600*l.* for 1 month; and conſequently 100*l.* multiplied by 9 months, muſt be equal to 900*l.* for 1 month: their ſeveral times therefore being reduced to one and the ſame time: their reſpective gains muſt bear the ſame proportion one to the other, as 600*l.* does to 900*l.* ſo that it is hereby reduced to a queſtion in Single Fellowſhip.

A and B in company: A put in the 1ſt of January 50*l.* but B could not put any money in till the 1ſt of May; what muſt B then put in to have an equal ſhare with A at the years end?

Anſwer 75*l.*

A, B and C in company; A put in the 1ſt of January, 100*l.* and the 1ſt of May 150*l.* more; and on the 1ſt of September takes out 30*l.* the remainder ſtays in till the year's end. B puts in the 1ſt of January 250*l.* and on the 1ſt of June 60*l.* more; and on the 1ſt of November 100*l.* more; which continues in till the year's end. C puts in the 1ſt of January 300*l.* and the 1ſt of April takes out 200*l.* and on the 1ſt of Auguſt takes out 50*l.* more: the remainder ſtays in till the years end. What muſt each have of the gain, which was 133*l.* *Anſwer.*

Anſwer A's part 40*l*. 14*s*. 2*d*. $\frac{1}{4}$ and ſomewhat over.
B's 64*l*. 12*s*. 4*d*. $\frac{3}{4}$ and ſomewhat over.
C's 27*l*. 1[illegible]*s* 4*d*. $\frac{3}{4}$ and ſomewhat over.

A, B and C in company, and put in together 3822*l*. A's money was in 3 months; B's money was in 5 months; and C's money was in 7 months; they gained 434*l*. which was ſo divided, as the $\frac{1}{2}$ of A's gain, was equal to $\frac{1}{3}$ of B's gain; and $\frac{1}{3}$ of B's gain was equal to $\frac{1}{4}$ of C's gain: what did each merchant gain and put in?

A gained 52*l*. B 78*l*. C 104*l*.

FIRST ſuppoſe A's gain to be any ſum (at pleaſure) to which you can proportion B's and C's according to the tenor of the queſtion: as for inſtance 4*l*. then of courſe B's muſt be 6*l*. and C's 8*l*. which being added together the total ſum is 18*l*. then ſay, As the total of thoſe ſeveral ſums, are to the ſum they ought to be (namely the whole gain) ſo will each particular ſum, be to the real part of the profit, which A ought to have, B ought to have, and C ought to have.

2d Multiply their ſeveral ſhares of the gain, by their reſpective times, and the proportion will be,

As the *ſum* of all thoſe products,
Is to the *whole ſtock*;
So will each *particular product*,
Be to each perſons *reſpective ſhare of the ſtock*.

Or thus. Divide the whole ſtock, by the ſum of all the afore-mentioned products; and by the quotient, multiply each particular product; ſo ſhall the ſeveral new products thus found, be the reſpective ſtocks of A, B, and C.

CHAP. XII.

Of EXCHANGE.

TO treat of the various rates of Exchange between England, and every other country, to which its commerce is extended, would be unneceſſary here, as there are ſeveral books wrote intirely on that ſubject; with decimal tables for the more readily finding what any particular ſum, in one country, is equal to in another country; at any certain rate of Exchange whatever.

BUT if the learner does not chuſe to depend on thoſe tables, (although they are very accurate) he may find the anſwer by *Reduction*, or the *Rule of Three*, which preſuming him by this time to be thoroughly acquainted with; I ſhall ſay no more on the ſubject.

The END *of* COMMON ARITHMETIC.

* * *

PART II.

OF

FRACTIONS,

Vulgar & Decimal.

Together with the USE and APPLICATION of them, in Calculating *Intereſt*, *Annuities*, *&c.*

CHAP. I.

Of VULGAR FRACTIONS.

NOTATION OF FRACTIONS depends upon this Suppoſition, "That one whole "Thing, whatſoever it be, may be diviſible into any number of equal parts:" —when no ſuch actual DIVISION can be made: ſuch as one thouſand part of a *grain*, a *farthing*, &c. for which part, or parts, we have no name, or denomination. A *Fraction*, or broken number, is one or more part, or parts of an Integer or whole thing, according as it is divided; and muſt conſiſt of two parts; viz a *Numerator*, and *Denominator*, which are placed one above the other, and ſeparated by a line thus, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{8}$, &c. and are to be expreſs'd in this manner: *One half*, *Three fourths*, *Five eights*, &c. The figure below the line is call'd the *Denominator*, and ſheweth into how many equal parts one Integer or unit is to be divided; and the figure above the line is call'd the *Numerator*, and ſheweth how many of the ſaid parts are contain'd in the Fraction,

A VULGAR FRACTION is either *Simple* or *Compound*; *Proper* or *Improper*.

A *Simple Fraction* is that which conſiſts of one *Numerator*, and one *Denominator*, as , $\frac{5}{12}$, $\frac{15}{24}$, &c.

A *Compound Fraction* is a Fraction of a Fraction, as $\frac{3}{5}$ of $\frac{3}{4}$, or $\frac{1}{3}$ of $\frac{6}{8}$, &c.

A proper Fraction, is that whoſe *Numerator*, is leſs than its *Denominator;* ſuch are the fractions beforementioned, viz. $\frac{3}{8}$, $\frac{5}{12}$, $\frac{15}{24}$, and the like.

AN *improper Fraction*, is that whoſe *Numerator* is either greater, or at leaſt equal to its *Denominator*; and ſheweth, that there are 1 or more integers contained in the Fraction. If the *Numerator* and *Denominater* of a fraction be the ſame, then is the fraction equal to 1 unit; but if the *Numerator* be greater than the *Denominator*; then ſo often as the ſaid *Numerator*, is contained in its *Denominator;* juſt ſo many integers will be found in the fraction; and the remainder (if any) being placed over the Diviſor, will be a fractional part of 1 of theſe integers;

So $\frac{4}{4}$ $\frac{9}{9}$ $\frac{28}{7}$ $\frac{427}{4}$ are all of them improper fractions; the 1ſt two of which being each of them equal to one unit, or integer; their *Numerator* and *Denominator* being equal.

S. How does a ſimple fraction ariſe?

M. From *Diviſion*, as will appear by the 17th example, page 73, where it is required to divide 427*l.* equally among 4 men: ſee the operation page 77. There you'll find after dividing 427 by 4, your 1ſt quotient is 106, which is that number of pounds to each man, and 3 for a remainder: which remainder is the *Numerator* of a *Simple Fraction*, whoſe *Denominator* is the *Diviſor*, which will ſtand thus $\frac{3}{4}$, and ſignifies three fourths of 1 unit in the *Quotient*: therefore each man will have 106*l.* and three fourths of a pound, to his ſhare: which being placed after this manner, 106 $\frac{3}{4}$ makes what is called a mixed number.

REDUCTION of VULGAR FRACTIONS.

SECTION 1ſt.

To Reduce a Fraction to its loweſt Terms.

Rule. Divide the *Denonominator* by the *Numerator*; and again the Diviſor by the remainder, and ſo proceed in that manner; ſtill dividing the laſt diviſor by the laſt remainder, till nothing remains; the laſt diviſor ſo found, will be the greateſt common meaſure which will divide both *Numerator* and *Denominator* of the given *Fraction*; after which, it will be in the loweſt terms, to which it is capable of being reduced. Note, if the laſt *Diviſor* be one, then was the *Fraction* in its loweſt terms at firſt.

EXAMPLES.

1ſt, Reduce $\frac{17136}{22848}$ to its loweſt terms.

HERE 5712 being the laſt diviſor, I therefore conclude it to be the greateſt common meaſure, or diviſor which will divide the *Numerator* and *Denominator* of the given *Fraction* without a remainder; the quotients of which, will be the new *Numerator* and *Denominator* of a *Fraction*, equal to the firſt *Fraction* given, and in its loweſt terms. As appears by the work,

17136)22848(1
17136
―――
5712(17136(3
17136
―――
.....

5712)17136(3 new Num.
17136
―――
.....

5712)22848(4 new Denom.
22848
―――
.....

Wherefore I have for a new *Fraction* $\frac{3}{4} = \frac{17136}{22848}$

2d. Reduce $\frac{17952}{22848}$ to its loweſt terms.

Diviſor 17952)22848(1
17952
―――
2d Diviſor 4896(17952(3
14688
―――
3d Diviſor 3264(4896(1
3264
―――
Laſt Div. or Remr. 1632(3264(2
3264
―――
....

1632)22848(14 new *Deno.*
1632
―――
6528
6528
―――
....

1632)17952(11 new *Num.*
1632
―――
1632
1632
―――
....

Wherefore I find that $\frac{11}{14}$ is the loweſt terms to which the *Fraction* $\frac{17952}{22848}$ can be reduced.

3d. Reduce $\frac{20160}{22848}$ to its loweſt terms. *Anſr.* $\frac{15}{17}$.
4th. Reduce $\frac{105}{135}$ to its loweſt terms. *Anſr.* $\frac{7}{9}$.
5th. Reduce $\frac{161}{184}$ to its loweſt terms. *Anſr.* $\frac{7}{8}$.
6th. Reduce $\frac{96}{130}$ to its loweſt terms. *Anſr.* $\frac{48}{65}$.

S. Is there no other way of reducing Fractions to their loweſt terms?

M. When the fractions given to be reduced, conſiſt of few figures: It may be done by dividing the *Numerator* and *Denominator*, by any one number which you can eſpy that will divide them both equally, whether it be the greateſt common Diviſor or not: and then again, divide their reſpective quotients by ſome other number, and ſo on, till you find it will divide no longer, and then it is in its loweſt terms: you may likewiſe often abreviate your fractions (when the *Numerator* and *Denominator* both end with a cypher, or cyphers) by cutting off an equal number in each.

S. Where is the neceſſity of reducing Fractions to their loweſt terms?

M. Becauſe they are fitter for uſe; being much eaſier handled than large numbers.

S. There is ſtill one thing more which I ſhould be glad to know: and that is, the reaſon on which the general rule is founded, by which we reduce Fractions to their loweſt terms?

M. To demonſtrate that, let us again take a view of the 2d example in this ſection, as it ſtands wrought there.

THERE you may obſerve, that the laſt *remainder* is contained in the 3d *Diviſor* } 3264 2 times.

And

	[*Brought forward*]	
And the 3d *Diviſor* being only once contained in its own *Dividend*, with the ſaid	3264	2 times
	Added	
laſt remainder over: it muſt therefore be contained in it ſelf - - - - - - -	1632	1 time
The ſum of which two muſt therefore be the third *Dividend* or ſecond *Diviſor*	4896	more
Then becauſe the ſaid 2d *Diviſor* (in which it was found 3 times) is likewiſe	‡ *And likewiſe*	
contained 3 times in its own	*Multiply*	3
Dividend: we therefore	*By*	3
multiply the 2d *Diviſor* by	3 ‡	
and the *product* is - -	14688 in which	9 times
To which if we add its own *remainder* which was the 3d *Diviſor* - - -	3264 in which	2 d
We have the 1ſt *Diviſor*, or *Numerator* of the fraction	17952 in which	11
So that we find 1632 to be contained in 17952 - -		11 d
To which if we add its own remainder, which was the 2d *Diviſor* - - -	4896 in which	3
The ſum will be the *Denominator* - - - -	22848 in which	14 d
Wherefore it is reduced to $\frac{11}{14}=\frac{17952}{22848}$ - - -		

SECTION 2d.

TO reduce fractions which have different denominators, to other fractions of the ſame value, which ſhall have a common *Denominator*, that is, each *Numerator* the ſame *Denominator*.

Rule. Multiply each *Numerator*, into all the *Denominators* but its own, for a new *Numerator*; and all the *Denominators* together for a common *Denominator*.

EXAMPLES.

1ſt Reduce $\frac{3}{4}$ and $\frac{5}{8}$ to a *Common Denominator.*

Anſwer $\frac{24}{32}$ and $\frac{20}{32}=\frac{3}{4}$ and $\frac{5}{8}$

2d Reduce $\frac{7}{9}$ and $\frac{12}{15}$ to a *Common Denominator.*

Anſwer $\frac{105}{135}=\frac{7}{9}$ and $\frac{08}{135}=\frac{12}{15}=\frac{4}{5}$

3d Reduce $\frac{6}{8}$, $\frac{9}{12}$, $\frac{11}{14}$ and $\frac{15}{17}$ to a *Common Deno.*

	6	8	11
	12	9	8
The operation	72	72	88
	14	14	12
	288	288	1056
	72	72	17
	1008	1008	7392
	17	17	1056
	7056	7056	3d 17952
	1008	1008	
1ſt *new Num.*	17136	2d 17136	

15	17
14	14
60	68
15	17
210	238
12	12
2520	2856
8	8
4th 20160	*Com. Denom.* 22848

$\frac{17136}{22848}=\frac{6}{8}$ or $\frac{3}{4}$ $\frac{17136}{22848}=\frac{9}{12}$ or $\frac{3}{4}$

$\frac{17952}{22848}=\frac{11}{14}$ $\frac{20160}{22848}=\frac{15}{17}$

No

Note. Thoſe examples and the 3 firſt in the laſt ſection prove each other.

S. I ſhould be obliged to you, if you would explain to me the reaſon why this operation ſhould produce new Fractions, equal to the firſt Fraction given.

M. I have already proved to you, that multiplying or dividing any 2 numbers by 1 and the ſame ſum, that the products or quotients ſo produced, ſhall have the ſame reaſon or proportion one to the other, as the 1ſt two numbers had before they were ſo multiplied or divided. By the aforegoing operation then, you may obſerve, that every *Numerator* and *Denominator* of the Fractions given to be reduced to a common *Denominator*, is multiplied each of them by the ſame numbers.

S. Is there no ſhorter way of finding a *Common Denominator?*

M. Sometimes it will happen, that by aſſigning ſome one of the *Denominators* of the Fractions given to be reduced for a common *Denominator*; you may find a new *Numerator* to each of the other Fractions, which ſhall have the ſame proportion to the aſſigned *Denominator*, as the *Numerators* of the given Fractions, had to their reſpective *Denominators*; but the inconvenience which would ariſe from practiſing this method, I ſhall ſhew in the next ſection, to which it more properly belongs.

Scho. Pray what is the uſe of reducing Fractions to a common *Denominator?*

Maſt. Becauſe when the ſeveral Fractions of the ſame Integer have equal *Denominators*; then are the parts of ſuch Fractions which are expreſs'd by the ſeveral *Numerators*, all of the ſame name or denomination; and therefore may be added together, or ſubtracted one from the other as the nature of the operation requires. As for inſtance if we were to add $\frac{1}{8}$ and $\frac{2}{8}$ of a pound Sterling, together, the product would be $\frac{3}{8}$ or which is the ſame thing, 5 half crowns: for if we divide a pound into 8 equal parts, each part will be exactly half a crown: conſequently two of thoſe parts muſt be two half crowns: and for the ſame reaſon the other Fraction muſt be 3 half crowns: whereas had they had different *Denominators*, viz. $\frac{3}{8}$ and $\frac{2}{5}$ they could not have been added, nor ſubtracted: for the $\frac{3}{8}$ would (as I ſhewed juſt now) been 3 half crowns, and the $\frac{2}{5}$ would have been two pieces of 4*s*. each: for a pound divided into 5 equal parts is 4*s* each, and conſequently 2 of thoſe parts, are 2 pieces of 4*s* each: ſo that being of different denominations (as obſerv'd before) they neither could be added nor ſubtracted. Here follows ſome more examples to exercife the learner.

1ſt. Reduce $\frac{3}{8}$ $\frac{5}{7}$ $\frac{4}{9}$ and $\frac{9}{10}$ to a common *Denom.*

Anſwer $\frac{1890}{5040}$ $\frac{3600}{5040}$ $\frac{2740}{5040}$ $\frac{4536}{5040}$

6th. Reduce $\frac{12}{15}$ $\frac{7}{19}$ $\frac{8}{15}$ & $\frac{14}{16}$ to a common *Deno.*

Anſwer $\frac{54720}{68400}$ $\frac{25200}{68400}$ $\frac{36480}{68400}$ $\frac{59850}{68400}$

7th. Reduce $\frac{13}{20}$ $\frac{24}{25}$ $\frac{30}{49}$ & $\frac{45}{60}$ to a common *Deno.*

Anſwer $\frac{255500}{1420000}$ $\frac{1411200}{1420000}$ $\frac{900000}{1420000}$ $\frac{110250}{142000}$

8th. Reduce $\frac{1}{2}$ $\frac{3}{4}$ $\frac{7}{8}$ $\frac{9}{12}$ & $\frac{12}{15}$ to a common *Deno.*

Anſwer $\frac{8640}{11520}$ $\frac{10080}{11520}$ $\frac{8640}{11520}$ $\frac{9216}{1152}$

SECTION 3d.

TO reduce a Fraction from one denomination to another of the ſame value, having the *Denominator* of the required Fraction given.

Rule. As the *Denominator* of the given Fraction,
Is to its *Numerator*;
So is the *Denominator* of the Fraction required;
To its *Numerator*.

EXAMPLES.

1ſt. Reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe *Denominator* ſhall be 8. *Anſwer* $\frac{6}{8}$.

2d. Reduce $\frac{7}{9}$ to a Fraction of equal value, whoſe *Denominator* ſhall be 15. *Anſwer* $\frac{11\frac{6}{9}}{15}$.

HERE according to the rule, I ſay,

deno *num* *deno*
If 4 : 3 :: 8
3
4) 24 (
6

which by the Rule of Three Direct, I find to be 6, inſtead of 3 for a *Numerator*, which will have 8, for its *Denominator*: ſo that if we conſider 8, (the aſſigned *Denominator* here) as the *Denominator* of 1 of the given Fractions in the 1ſt example, in the laſt ſection; and the Fraction here propoſed to be reduced, viz. $\frac{3}{4}$, to be the other Fraction; it will appear from the above operation, that by this method the afore-mentioned Fractions may be reduced to a common *Denominator*, and in lower terms than by the general rule; for by the above method we have $\frac{6}{8}$ and $\frac{5}{8}$, whereas by the common way of reducing them, we have $\frac{24}{32}$ and $\frac{20}{32}$.

LET us now try if the two fractions in the 2d example in the laſt ſection can be reduced to a common *Denominator* by the forgoing method, by working the 2d example in this ſection, which is exactly ſimilar to it.

deno *num* *deno*
If 9 : 7 :: 15
7
) 105 (
$11\frac{6}{9}$.

THIS proves the inconvenience of finding a common *Denominator* by this method; as the new *Numerators* ſought, will not always conſiſt of whole numbers; for in this example, inſtead of $\frac{7}{9}$, we want a new Fraction equal to it, whoſe *Denominator* ſhall be 15. The *Numerator* therefore which bears the ſame proportion to 15 as 7 does to 9; is found by the foregoing operation to be $11\frac{6}{9}$, which forms a new Fraction called a mixt Fraction, and would ſtand thus $\frac{11\frac{6}{9}}{20}$

More Examples for the Exerciſe of the Learner.

3d Reduce $\frac{7}{8}$ to a Fraction of the ſame value, whoſe *Denominator* ſhall be 23.

Anſwer $\frac{20\frac{1}{8}}{23}$

4th Reduce $\frac{3}{10}$ to a Fraction of the ſame value, whoſe *Denominator* ſhall be 40.

Anſwer $\frac{12}{40}$.

SECTION 4th.

TO reduce a Fraction from one *Denomination* to another of the ſame value, having the *Numerator* of the required Fracton given.

Rule. As the *Numerator* of the given Fraction,
Is to its *Denominator*:
So is the *Numerator* of the required Fraction,
To its *Denominator*.

EXAMPLES.

1ſt Reduce $\frac{6}{8}$ to a Fraction of the ſame value, whoſe *Numerator* ſhall be 3.

Anſwer $\frac{3}{4}$.

See the Operation.

$$\begin{array}{ccccc} \textit{num.} & & \textit{denom.} & & \textit{num.} \\ \text{If } 6 & : & 8 & :: & 3 \\ & & & & 8 \\ \hline & & & 6) & 24\ (\\ \hline & & & & 4 \end{array}$$

2d Reduce $\frac{11\frac{6}{9}}{15}$ to a fraction of the ſame value, whoſe *Numerator* ſhall be 7.

Anſwer $\frac{7}{9}$.

3d Reduce $\frac{20\frac{1}{8}}{23}$ to a Fraction of the ſame value, whoſe *Numerator* ſhall be 7.

Anſwer $\frac{7}{8}$.

4th. Reduce $\frac{30}{40}$ to a Fraction of the ſame value, whoſe *Numerator* ſhall be 9. *Anſwer* $\frac{9}{12}$

Note. *The queſtions in this ſection prove thoſe in ſection 3d; but before yon can work the 2d and 3d. I muſt ſhew you how to reduce a mixt Fraction to a ſimple or ſingle Fraction.*

SECTION 5th.

TO reduce a mixt *Fraction* to a ſingle one.

Rule. If the Fractional part belong to the *Numerator*; then multiply ſuch *Numerator*, by the *Denominator* of the Fractional part taking in its *Numerator* for a new *Numerator*; and likewiſe multiply the *Denominator* of the Fraction, by the *Denominator* of the Fractional part for a new *Denominator*; and contrariwiſe when the Fractional part belongs to the *Denominator*: always remembering to add the *Numerator* of the Fractional part, to the *Numerator*, or *Denominator*, to which it is anext.

EXAMPLES.

1ſt. Reduce $\frac{11\frac{6}{9}}{15}$ to a ſimple Fraction, *Anſr.* $\frac{105}{135} = \frac{7}{9}$

2d Reduce $\frac{20\frac{1}{8}}{23}$ to a ſingle Fraction. *Anſr.* $\frac{161}{184} = \frac{7}{8}$.

Operation of 1ſt. $11\frac{6}{9}$ 15 Operation of 2d. $20\frac{1}{8}$ 23

9 9 8 8

—— —— —— ——

N. 105 *D.* 135 *N.* 161 *D* 184

3d Reduce $\frac{12}{16\frac{1}{4}}$ to a ſingle Fraction. *An.* $\frac{96}{130} = \frac{48}{65}$.

4th. Reduce $\frac{25}{36\frac{3}{4}}$ to a ſingle Fraction. *Anſr.* $\frac{100}{147}$

Note 1ſt. *The 1ſt and 2d examples above, prove the 2d and 3d example in ſection 4th.*

Note 2d. *In the 1ſt and 2d examples above, the* Numerator *of the Fractional part is to be taken in when you multiply the* Numerator; *and in the 3d and 4th examples, when you multiply the* Denominator.

SECTION 6th.

TO reduce a mixt number to an improper *Fraction.*

Rule. Multiply the whole number by the *Denominator* of the Fraction, and to its product add the *Numerator,* for a new *Numerator;* whoſe *Denominator* ſhall be that of the Fraction.

Note 1ſt. *To expreſs a whole number, Fraction-wiſe, put* 1 *for its* Denominator.

EXAMPLES.

1ſt. Reduce $106\frac{3}{4}$ to an improper Fraction.

106 $\frac{3}{4}$

4

—— *Anſwer* 427 *equal to the given mixt number.*

$\frac{427}{4}$

2d. Reduce $43\frac{5}{8}$ to an improper Fraction. *Anſr.* $\frac{349}{8}$

3d. Reduce $19\frac{12}{18}$ to an improper Fraction. *Anſr.* $\frac{354}{18}$

4th. Reduce $24\frac{105}{100}$ to an improper Fraction. *Anſr* $\frac{2505}{100}$

5th. Reduce $30\frac{10}{15}$ to an improper Fraction. *Anſr.* $\frac{460}{15}$

6th. Reduce $45\frac{35}{37}$ to an improper Fraction. *Anſr.* $\frac{1700}{37}$

Note 2d. *When the Fractional part of any mixt number is ſuch as you can ſee will reduce to lower terms (as in the 3d and 5th examples in this ſection) then take the Fraction ſo reduced, inſtead of the firſt Fraction given and work as before directed; and the improper Fraction ſo found, will be equal to the other improper Fraction. So in the 3d example aforegoing* $\frac{12}{18}$ *will be reduced to* $\frac{2}{3}$ *and ſo will* $\frac{10}{15}$ *in the 5th example: which being each reduced to their improper Fraction will be* $\frac{59}{3}$ *and* $\frac{92}{3}$ *equal in value to* $\frac{354}{18}$ *and* $\frac{460}{15}$ *as I ſhall prove in the next ſection.*

THE reaſon on which this operation is founded is this.

THE *Denominator* of the Fraction annexed to the whole number or numbers, ſhewing how many parts one Integer is to be divided into; which in the 1ſt example in this ſection is 4, we therefore multiply the Integers by 4, and take in the *Numerator* of the Fraction, and the product is then ſo many 4th parts; and the *Denominator* 4 being equal to 1 whole Integer, we therefore place it below.

S. What is the uſe of reducing mixt numbers to improper fractions?

M. The uſe of it is, that when a whole number, and a mixt number; a whole number and a Fraction; or a Fraction and a mixed number, are given to be multiplied, or divided one by the other; the operation cannot be performed fractionally, except they be both fractionally expreſs'd.

SECTION 7th.

TO reduce an improper Fraction to its whole, or mixt number,

Rule. Divide the *Numerator* by the *Denominator*; if nothing remains, then is the quotient the anſwer in whole numbers

IF any thing remains, place ſuch remainder after the quotient for a *Numerotor*; and the Diviſor under it for a *Denominator*: which will be a mixt number equal to the improper Fraction given.

EXAMPLES.

1ſt Reduce $\frac{427}{4}$ to a whole or mixed nnmber.

Anſwer 106 $\frac{1}{4}$, as will appear by the work.

$$4)\ \underline{427}\ ($$

$$106\frac{1}{4}$$

2d Reduce $\frac{349}{8}$ to a whole or mixed number.
Anſwer $43\frac{5}{8}$

3d Reduce $\frac{354}{18}$ to a whole or mixed number.
Anſwer 19 $\frac{12}{18}$

4th Reduce $\frac{2721}{109}$ to a whole or mixt number.
Anſwer 24 $\frac{105}{109}$.

5th Reduce $\frac{460}{15}$ to a whole or mixt number.
Anſwer 30 $\frac{10}{15}$.

6th Reduce $\frac{1700}{37}$ to a whole or mixt number.
Anſwer 45 $\frac{35}{37}$.

7th Reduce $\frac{108}{4}$ to a whole or mixt number.
Anſwer 27.

8th Reduce $\frac{59}{3}$ to a whole or mixt uumber.
Anſwer 19 $\frac{2}{3}=\frac{12}{18}$.

9th Reduce $\frac{92}{3}$ to a whole or mixt number.
Anſwer 30 $\frac{2}{3}=\frac{10}{15}$. *Note,*

Note. *The* 1ſt, 2d, 3d, 4th, 5th and 6th *examples in this ſection prove thoſe in the laſt ſection; and the* 3d *and* 8th *in this ſection being compared together, and the* 5th *and* 9th *together, will prove the obſervation, in the* 2d *note of the laſt ſection.*

SECTION 8th.

TO reduce a compound Fraction, to a ſingle or ſimple one.

Rule. Multiply all the *Numerators* together for a new *Numerator*, and all the *Denominators* together for a new *Denominator*.

EXAMPLES.

1ſt Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ to a ſimple Fraction.
See the work.

2	3
3	4
6	12
5	8
New Numer. 30	*New Denom.* 96

$\frac{30}{96}=\frac{5}{16}=$ the compound Fraction given.

2d Reduce $\frac{7}{9}$ of $\frac{8}{10}$ of $\frac{12}{15}$ to a ſimple Fraction.
Anſwer $\frac{672}{1350}$.

3d Reduce $\frac{12}{16}$ of $\frac{15}{19}$ of $\frac{25}{30}$ to a ſimple Fraction.
Anſwer $\frac{4500}{9120}$.

4th Reduce $\frac{7}{13}$ of $\frac{9}{25}$ of $\frac{5}{18}$ of $\frac{12}{20}$ to a ſimple Fraction *Anſwer* $\frac{3780}{117000}$.

5th Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{8}$ of $\frac{7}{9}$ to a ſimple Fraction.
Anſwer $\frac{210}{864}$.

AS I have ſhewed in the beginning of this chapter what a compound Fraction is, and how it ariſes; I think it needleſs to repeat it again in this place.

S. Still I cannot conceive why this operation ſhould produce a ſimple Fraction, equal to the compound one given.

M. I ſhall endeavour to make this plain unto you, by the following example. Suppoſe it were required to reduce $\frac{2}{3}$ of ^{1} to a ſimple Fraction; then by the foregoing rule it will be $\frac{2}{60}$: now if we ſuppoſe the compound Fraction to be that of 1*l.* ſterling; Then it is plain that $\frac{1}{20}$ is 1*s.* which again being divided into 3 parts, is 3 groats; two of which parts being taken, muſt be equal to 8*d.* wherefore $\frac{2}{3}$ of a ſhilling is equal to 8*d.*

BUT this is only the ſimple Fraction of a ſhilling, whereas the ſimple Fraction of a pound is required, which ſhall be equal to the compound Fraction given. Granted: but if we multiply the *Denominator* of the 1ſt Fraction, viz. 3 (which is the groats in 1 ſhilling) by the laſt *Denominator* 20, then it will give us the groats in a pound, viz. 60. *Now here obſerve well: that although inſtead of* 3, *we have* 60 *for a* Denominator, *yet are they parts of the ſame denomination as before; namely groats. Conſequently* $\frac{2}{60}$ *of a pound which is* 2 *groats: muſt be equal to* $\frac{2}{3}$ *of a ſhilling, it being alſo two groats.*

Scho. This definition indeed is very plain, where the *Numerator* of the laſt Fraction happens to be an unit; but ſuppoſe it had been any other number by which (according to the given rule) the *Numerator* of the other Fraction could have been multiply'd, as for inſtance 3.

M. This could in no wiſe alter the property; for as I have already proved that $\frac{2}{60}$ of a pound is equal to $\frac{2}{3}$ of a ſhilling, each being equal to eight pence; the

Numerator then of the laſt Fraction being three ſhillings; conſequently the number of parts taken muſt be 3 times more; wherefore multiplying the *Numerator* of the 1ſt Fraction, viz. 2, by 3, the *Numerator* of the laſt Fraction, we have $\frac{6}{20}$=6 times 4 pence, or 3 times 2 four pences = 2 ſhillings.

S, Or if the *Denominator* (inſtead of 20) had been ſuch, as would not have divided the Integer into any equal number of known parts, ſuch as crowns, half crowns, ſhillings, &c.

M. What-ever the *Denominator* had been, the proportion would ſtill have been the ſame. For inſtance let us ſuppoſe $\frac{2}{3}$ of $\frac{9}{13}$ of a pound. Here you ſee, that the laſt *Denominator* being 13, conſequently the integer, 1 pound, is ſuppoſed to be divided into 13 equal parts, and the *Numerator* ſhews that 9 of ſuch parts are contained in the Fraction: *Now if we ſuppoſe* 1 *of thoſe parts, to be equal to* 3 *of a leſſer* denomination, *it is, then plain that when we multiply* 3 *by* 13 *the product will be the number of thoſe leſſer parts contained in* 13, *viz.* 39; *wherefore* $\frac{2}{39}$ *of the whole integer, muſt be equal to* $\frac{2}{3}$ *of one of thoſe parts into which the whole is divided, namely,* $\frac{9}{13}$. *But* $\frac{2}{3}$ *of* $\frac{9}{13}$ *was required; therefore multiplying* 2 *by* 9, *gives us* $\frac{18}{39}$, *which is the fraction ſought.*

THE uſe of reducing compound Fractions to ſimple ones is evident, as we can neither add, ſubtract, multiply or divide, till they are ſo reduced.

SECTION 9th.

TO reduce a Fraction of one denomination to the Fraction of another, but greater, retaining the ſame value.

Rule. Reduce the given Fraction to a compound Fraction, by making it a Fraction of the ſeveral integers between it and that denomination you would reduce it to.

2d Reduce that compound Fraction to a ſingle one, by the rule in ſection 8th.

Or which is the ſame thing, multiply at once, the *Denominator* of the given Fraction by that number of parts of the ſame denomination of itſelf, which make one integer of that denomination you would reduce it to, the *Numerator* of which, will be that of the given Fraction.

EXAMPLES.

1ſt Reduce of a penny, to the Fraction of a pound.

Anſwer $\frac{3}{1200}=\frac{1}{400}$.

2 Reduce $\frac{1}{2}$ of a farthing to the Fraction of a ſhilling

Anſwer $\frac{7}{96}$.

3d Reduce $\frac{7}{6}$ of an ounce Troy, to the Fraction of a pound.

Anſwer $\frac{7}{96}$.

4th Reduce $\frac{5}{7}$ of a *lb.* Avoirdupoiſe, to the Fraction of a *cwt.*

Anſwer $\frac{5}{784}$.

6th Reduce $\frac{9}{13}$ of a pint of wine, to the Fraction of a *hhd.*

Anſwer $\frac{6}{8352}$.

THE work of the 1ſt queſtion will ſtand thus, firſt will be $\frac{3}{5}$ of $_{12}$ of $\frac{1}{2}$

	3		5	
	1		12	New Fraction $\frac{1}{400}$
	3		60	
	1		20	
Numer.	3	Denom.	1200	

Or according to the 2d method thus

$$\frac{\begin{matrix}5\\240\end{matrix}}{1200} \qquad \frac{3}{1200}\ \text{as before.}$$

S. What is the uſe of this rule?

M. Becauſe in the Single Rule of Three, in Fractions, (as in whole numbers) your 1ſt and 3d terms muſt be of the ſame denomination; and in the Double Rule, your 1ſt and 4th, and 2d and 5th. Therefore when theſe terms happen to be of different denominations, they muſt be reduced to Fractions of the ſame denomination; and ſo muſt they likewiſe be, when Fractions of different denominations are given to be added together or ſubtracted one from the other.

SECTION 10th

TO Reduce a Fraction of one denomination to the Fraction of another, but leſſer, retaining the ſame value.

Rule. Multiply the given *Numerator*, by the parts of the denominations between it and that denomination you would reduce it to, for a new *Numerator*, and place it over the given *Denominator*.

EXAMPLES.

1ſt Reduce $\frac{3}{1200}$ of a pound to the Fraction of a penny. *Anſwer* $\frac{3}{5}$.

2d Reduce $\frac{1}{96}$ of a ſhilling to the Fraction of a farthing. *Anſwer* $\frac{1}{2}$.

3d Reduce $\frac{7}{96}$ of a *lb.* troy, to the Fraction of an ounce. *Anſwer* $\frac{7}{8}$.

4th Reduce $\frac{5}{784}$ of a *cwt.* to the Fraction of a *lb.* *Anſwer* $\frac{5}{7}$.

5th Reduce $\frac{9}{6532}$ of a *hhd.* of wine, to the Fraction of a pint. *Anſwer* $\frac{9}{3}$.

I ſhall here ſhew you bow the 1ſt example is wrought, by which you may work all the reſt First then reduce it to a compound Fraction thus, $\frac{3}{1200}$ of a pound, is $\frac{3}{1200}$ of $\frac{20}{1}$ of $\frac{12}{1}$.

$$\begin{array}{r} 3 \\ 20 \\ \hline 60 \\ 12 \\ \hline \end{array}$$

Numerator 720

Anſwer $\frac{720}{1200}$ which being reduced to its loweſt terms is $\frac{3}{5}$

Note. *The examples in this ſection and thoſe in ſection* 9th, *prove each other.*

SECTION 11th.

TO find the value of any Fraction in the known parts of an integer.

Rule. Multiply the *Numerator* by the common parts of the integer, and divide by the *Denominator*.

EXAMPLES.

1ſt. Reduce $\frac{3}{4}$ of a pound ſterling to its proper quantity.
Anſwer 1 *s*

2d Reduce $\frac{2}{3}$ of a pound *ſter.* to its proper quantity.
Anſwer 13*s.* 4*d.*

3d Reduce $\frac{18}{43}$ of a ſhilling to its proper quantity.
Anſwer $5^d\frac{1}{43}$

4th. Reduce $\frac{7}{9}$ of 4*l.* 10*s.* to its proper quantity.
Anſwer *l.* 10*s.*

5th. Reduce $\frac{12}{16}$ of a pound *Troy* to its proper quantity.
Anſwer 9 *oz.*

6th. Reduce $\frac{12}{60}$ of a *ton wt.* to its proper quantity.
Anſwer 4 *Cwt.*

7th. Reduce $\frac{7}{9}$ of a *lb Avoirdupoiſe* to its proper quantity.
Anſwer 12*lb* 18*dwt.* 21*gr* $\frac{3}{9}$

8th. Reduce $\frac{9}{10}$ of 12*c.* 1*qr.* 12*lb.* to its proper quantity.
Anſwer 11 *cwt.* 0*qrs* 13*lb.* 9*oz.* 12*dwts.*

9th. Reduce $\frac{4}{7}$ of a mile to its proper quantity.
Anſwer 4 *fur.* 125 *yds.* 2 *feet* 1 *in* $\frac{5}{7}$

10th. Reduce $\frac{9}{10}$ of a yard to its proper quantity.
Anſwer 2 *feet.* 8 *in* 1*bc.* $\frac{2}{10}$

11th. Reduce $\frac{4}{5}$ of an ell to its proper quantity.
Anſwer 1 *yard.*

12th. Reduce $\frac{7}{16}$ of an acre to its proper quantity.
Anſwer 1 *rood,* 30 *perches.*

13th. Reduce $\frac{4}{9}$ of a tun of wine to its proper quantity.
Anſwer 1 *hhd* 49 *galls.*

14th. Reduce $\frac{7}{8}$ of a barrel of beer to its proper quantity.
Anſwer 30 *galls.* $\frac{1}{2}$

15th. Reduce $\frac{3}{8}$ of a chaldron of coals to its proper quantity.
Anſwer 13 *buſh.* $\frac{1}{2}$

16th. Reduce $\frac{2}{7}$ of a quarter of corn to its proper quantity.
Anſwer 2 *buſh.* 1 *peck* $\frac{1}{7}$

17th. Reduce $\frac{7}{13}$ of a day natural to its proper quantity.
Anſwer 12 *hrs.* 55 *min* 23 *sec.* $\frac{1}{13}$

18th. Reduce $\frac{4}{5}$ of a month to its proper quantity.
Anſwer 3 *weeks,* 1 *day,* 9 *hrs* 36 *min.*

19th. What is the proper quantity of $\frac{7}{8}$ yard of cloth.
Anſwer 3 *qrs.* 2 *nails.*

20th What is the proper quantity of $\frac{2}{9}$ of a *hhd.* of beer.
Anſwer 12 *galls.*

Here follows the 2d and 4th example aforegoing wrought.

2d

```
2d example  2          4th example  £   s
           20                        4  10
           —                         20
         3)40(13s.                  ——
           3                         90
           —                          7
           10                       ——(2|0
            9                      9)630(7|0
           —                         63   ——
            1                       ——£3 : 10
           12                        ..0
          ——
         3)12(4 pence
           12
          ——
           ..
```

IN the 2d example, I multiply the *Numerator* 2 by 20, and divide by the *Denominator* 3, which gives me 13 ſhillings and 1 over; which 1 I again multiply by 12, and divide that product again by the *Denominator*, which gives 4*d* ſo that the value of $\frac{2}{3}$ of a pound ſterling I find to be 13*s.* 4*d* And in the 4th example I reduce the pounds and ſhillings into ſhillings, and then work as in the 2d example.

Scho I think (if I remember right) you told me ſomewhere or other before; that to find the value of a proper ſimple Fraction, was to divide the Integer into ſo many parts as the *Denominator* conſiſted of; as for inſtance, in the 2d example which was $\frac{2}{3}$ of a pound. the pound divided into 3 equal parts would be 6*s.* 8 pence each part. therefore 2 of thoſe parts would be 13*s.* and 4 pence as above.

Maſt. As you had not then been ſhewn how to reduce them properly, I could not ſo well have made myſelf underſtood otherwiſe, altho' you ſee it anſwered the ſame purpoſe; but then it could only be done, when the *denominator* happen'd to be ſuch a number as would divide the ſhillings, or pence in a pound, without a remainder; and the quotient been ſome known part of a pound without any Fractional part (which is not always the caſe) but this method of finding the value of a Fraction will hold good in all caſes whatever, and the Fraction (if any remains) at laſt will only be a Fraction of the loweſt name or denomination into which we can divide an Integer and beſides the operation is in effect the ſame: for whether we divide any of the known parts contain'd in an Integer, by the *Denominator*, and multiply the quotient by the *Numerator* (as by the firſt method) or multiply the *Numerator* by ſuch number of parts, and divide that product by the *Denominator*, it produces the very ſame anſwer.

THE uſe of this rule is, that when you have perform'd the operation of any queſtion by Fractions; you then by this find the value of the Fractional part of the anſwer; in the known parts of *money*, *weight*, *meaſure*, &c.

SECTION 12th.

TO reduce any given quantity, to the Fraction of any greater denomination of the ſame kind.

Rule. Reduce the given quantity to the loweſt terms mentioned for a *Numerator*; and the Integer into the ſame denomination, for a *Denominator*.

Note 1ſt. *If there be a Fraction in the given quantity ſuch as* 12*s.* 4*d* $\frac{3}{4}$ $\frac{1}{3}$

1ſt. Reduce your 12*s.* 4*d* $\frac{3}{4}$ into farthings, then multiply again by 3, and take in your 1, and the product will be ſo many thirds of a farthing, for a *Numerator*.

2d. Reduce the Integer into thirds of a farthing, for a *Denominator*.

EXAM.

EXAMPLES.

1ſt. Reduce 15*s.* to the Fraction of a pound ſterling. *Anſwer* $\frac{3}{4}$.

2d. Reduce 13*s.* 4*d.* to the Fraction of a pound. *Anſr* $\frac{2}{3}$

3d. Reduce 5*d* $\frac{1}{13}$ to the Fraction of a ſhilling. *Anſr* $\frac{11}{26}$

4th. What part of 4*l.* 10*s.* is 3*l.* 10*s.* *Anſr.* $\frac{7}{9}$.

5th. Reduce 9*oz.* to the Fraction of a *lb. Troy*. *Anſr* $\frac{9}{12}$

6th. Reduce 4*Cwt.* to the Fraction of a tun. *Anſr.* $\frac{1}{5}$

7th. Reduce 12*lb.* 18*dwt.* 21*gr.* $\frac{3}{5}$ to the Fraction of a *lb. Avoirdupoiſe.* *Anſwer* $\frac{7}{9}$.

8th. What part of 12*cwt.* 1*qr* 12*lb.* is 11*cwt.* 0*qr.* 13*lb.* 9*oz.* 12 *dwts.* $\frac{3}{5}$. *Anſwer* $\frac{9}{10}$

9th. Reduce 4 *furlongs*, 125 *yards*, 2 *feet*, 1 *inch* $\frac{5}{7}$ to the Fraction of a mile. *Anſwer* $\frac{4}{7}$.

10th. Reduce 2 *feet*, 8 *in.* 1*bc.* $\frac{2}{10}$ to the Fraction of a yard. *Anſwer* $\frac{9}{10}$.

11th. Reduce 1 *yard* to the Fraction of an ell. *Anſr* $\frac{4}{5}$

12th. Reduce 1 *rood*, 30 *poles*, to the Fraction of an acre. *Anſwer* $\frac{7}{16}$.

13th. Reduce 1 *hhd.* 49 *galls.* of wine to the Fraction of a tun. *Anſwer* $\frac{4}{9}$ *tun.*

14th. Reduce 31 *galls* $\frac{1}{2}$ of beer to the Fraction of a barrel. *Anſwer* $\frac{7}{8}$ *barrel.*

15th. Reduce 13 *buſhels* $\frac{1}{2}$ of coals to the Fraction of a chaldron. *Anſwer* $\frac{3}{8}$ *chaldron.*

16th. Reduce 2 *buſhels* 1 *peck*, $\frac{1}{7}$ of corn to the Fraction of a quarter. *Anſwer* $\frac{2}{7}$ *quarter.*

17th. Reduce 12 *hrs.* 55 *min.* 23 *ſec.* $\frac{1}{13}$ to the Fraction of a day natural. *Anſwer* $\frac{7}{13}$

18th. Reduce 3 *wks.* 1 *day*, 9 *hrs*, 36 *min.* to the Fraction of a month. *Anſwer* $\frac{4}{5}$.

Note; *The example in this ſection, and thoſe of the laſt prove each other.*

THIS rule teacheth you (when the terms of any queſtion propoſed are given in any of the known parts of an Integer) how to reduce ſuch parts to the Fraction of that Integer; in order to perform the operation Fractionally.

CHAP. II.

ADDITION *of* VULGAR FRACTIONS.

WHEN the numbers given are ſingle Fractions, and have equal denominators.

Rule. Add all the *Numerators* together, for a new *Numerator*, whoſe *Denominator* ſhall be be the common *Denominator*.

Note. *If after the* Numerators *are added together, the ſum ſhould be greater than the* Denominator, *then reduce ſuch improper Fraction to its whole or mixt number, by the rule given in the* 7th *ſection of the laſt chapter.*

EXAMPLES.

1ſt Add $\frac{2}{8}$, $\frac{3}{8}$, and $\frac{1}{8}$ together.

$$\begin{array}{r} 2 \\ 3 \\ 1 \\ \hline 6 \end{array}$$

Anſwer $\frac{6}{8} = \frac{3}{4}$.

2d Add $\frac{7}{9}$, $\frac{5}{9}$, $\frac{3}{9}$, and $\frac{2}{9}$ together.

$$\begin{array}{r} 7 \\ 5 \\ 3 \\ 2 \\ \hline 17 \end{array}$$

Anſwer $\frac{17}{9} = 1\frac{8}{9}$

SECT.

SECTION 1ſt.

WHEN the Fractions given to be added have unequal *Denominators*.

Rule. Reduce them to one common *Denominator*, by the rule given in the 2d ſection of the laſt chapter, and then proceed to add them according to the 1ſt rule in this chapter.

EXAMPLES.

1ſt Add $\frac{2}{3}$, $\frac{3}{4}$, $\frac{6}{8}$ together

2	3	6	8
4	3	4	4
8	9	24	32
8	8	3	3
1ſt *Nu.* 64	2d *Nu.* 72	3d *Nu.* 72	*co. De.* 96

64
72
72
208 ſum of all the *Numerators*.
96 common *Denominator*.

which being reduced to its proper terms, will be found equal to 2 $\frac{16}{96}$ or 2 $\frac{1}{6}$.

SECTION 2d.

WHEN any of the Fractions given to be added is a compound Fraction.

Rule. Such compound Fraction or Fractions muſt firſt be reduced to a ſimple one, and then proceed according to the rule in the laſt ſection.

EXAMPLE.

1ſt Add $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{5}{8}$, of $\frac{1}{7}$ together.

1ſt I reduce $\frac{5}{8}$ of $\frac{3}{7}$ to a ſimple Fraction, which is $\frac{15}{56}$. 2d I reduce $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{15}{56}$ to a common *Denominator*. And laſtly, I add all the *Numerators* together, and under the ſum place the common *Denominator*

3	2	15
3	4	3
9	8	45
56	56	4
1ſt *Num.* 504	2d *Num.* 448	3d *Num.* 180

56		504
3	Thus	448
		180
168		
4	Sum of the *Numerators*	1132
Com. *Deno.* 672	Com. *Deno.*	672

WHICH being reduced to its proper terms is $1\frac{460}{672} = \frac{115}{168}$.

SECTION 3d.

WHEN Fractions of integers of different denominations are given to be added together.

Rule. Reduce them all to Fractions of one and the ſame integer, by the rule given in the 9th or 10th ſection in the laſt chapter: if to the Fraction of a greater denomination, by the 9th: but if to that of a leſſer denomination, by the 10th, and then add them together, as before directed.

EXAMPLES.

1ſt Add $\frac{7}{9}$ of a pound ſterling, $\frac{3}{5}$ of a ſhilling and $\frac{2}{3}$ of a penny together

TO perform this queſtion, I firſt make $\frac{3}{5}$ of a ſhilling, and $\frac{5}{8}$ of a penny a compound Fraction of a pound, thus. Firſt I ſay, $\frac{3}{5}$ of a ſhilling, is $\frac{3}{5}$ of $\frac{1}{20}$, and $\frac{5}{8}$ of a penny, is $\frac{5}{8}$ of $\frac{1}{12}$ of $\frac{1}{20}$, which compound Fractions being reduced to ſimple ones, by the rule in the 8th Section of the laſt chapter. I have $\frac{3}{100}$ and $\frac{5}{1920}$ of a pound: which together with $\frac{7}{9}$ of a pound being reduced to a common *Denominator* by the 2d ſection of the laſt chapter, we have the

New Fractions $\frac{51840}{1728000}$ $\frac{4500}{1728000}$ $\frac{1344000}{1728000}$.

I next add them together, according to the 1ſt ſection of this chapter, and find the ſum to be $\frac{1400340}{1728000}$ which being reduced to its proper value according to the rule in the 11th ſection of the laſt chapter, I find equal to 16*s*. 2*d* $\frac{1}{4}$ $\frac{8352}{8640}$.

Note. *I have not in the above example given you the full operation; but I have referred you to the rules given in the ſeveral ſections, by which they are performed: but as the* 2d *example is not ſo long and tedious, I ſhall give you the work at large.*

2d Add $\frac{2}{3}$ of a pound troy, to $\frac{3}{4}$ of an ounce. Performed thus.

Firſt I reduce the $\frac{3}{4}$ of an ounce to the Fraction of a pound, by ſaying $\frac{3}{4}$ of an ounce is $\frac{3}{4}$ of $\frac{1}{12}$ $=\frac{3}{48}$ of a *lb* then placing the $\frac{2}{3}$ of a pound before it thus $\frac{2}{3}$ $\frac{3}{48}$ I reduce them to a common *Denominator.*

Numerator $\frac{96}{144}$ $\frac{9}{144}$ 144 *Com. Denom.*

Which two *Numerators* being added, and their ſum placed over the common *Denominator* will ſtand thus;

$$\begin{array}{r} 96 \\ 9 \\ \hline 105 \\ \hline 144 \end{array}$$

which by the 11th ſection in the laſt chapter will be found equal to 8 ounces. 15 penny weights.

SECTION 4th.

WHEN mixt numbers are given to be added which are of the ſame denomination.

Rule. Firſt find the ſum of the Fractions by the 1ſt or 2d rule given in the 1ſt ſection in this chapter; then add the integer or integers, (if there be any found) in the ſum of the Fraction, to the whole numbers, and the Fraction (if any) annex to the total ſum of the whole numbers, which will be the anſwer.

EXAMPLES.

1ſt Add 12 $\frac{5}{8}$, 15$\frac{3}{4}$ and 9 $\frac{7}{10}$ of a pound ſterling, together.

Firſt then I reduce $\frac{5}{8}$, $\frac{3}{4}$, and $\frac{7}{10}$ to a common *Denominator*.

5	3	7	10
4	8	4	4
20	24	28	40
10	10	8	8
1ſt *Nu.* 100	2d *Nu.* 240	3d *Nu.* 224	c. *Den*: 320
320	320	320	

THEN adding all the *Numerators* together and placing below the total ſum, the common *Denominator*, I find the ſum of all the fractions $= \frac{564}{320}$ of a pound, which being an improper Fraction, I find the value of it according to the rule in the 7th ſection of the laſt chapter, thus.

$$320)564(1l.\ \tfrac{244}{320} = \tfrac{61}{80}$$
$$\underline{320}$$
$$244$$

I therefore add the Integer ſo found, to the Integers given in the queſtion; and to the total ſum annex the Fraction, in this manner.

$$\begin{array}{r} 12 \\ 15 \\ 9 \\ \underline{1} \\ 37\ \tfrac{344}{320} \cdot \tfrac{61}{80} \end{array}$$

which Fraction being valued according to the rule in the 11th ſection of the laſt chapter, will be found equal to 15*s.* 3*d.* ſo that the whole amount of the mixt numbers given to be added is 37*l.* 15*s.* 3*d.*

More examples for the exerciſe of the learner.

1ſt. Add $\frac{1}{2}$ and $\frac{7}{8}$ together. *Anſwer* 1 $\frac{6}{16}$.
2d. Add $\frac{7}{10}$ and $\frac{11}{12}$ and $\frac{4}{9}$ together. *Anſwer* 2 $\frac{66}{1080}$
3d. Add 19 and 7 $\frac{1}{2}$ of $\frac{2}{3}$ together. *Anſwer* 26 $\frac{3}{8}$
4th. Add $\frac{1}{2}$ of $\frac{7}{8}$ and $\frac{2}{3}$ of $\frac{19}{20}$ together. *Anſwer* $\frac{68}{980}$
5th. Add $\frac{1}{3}$ of 95 and $\frac{7}{8}$ of 14 together. *Anſr.* 43 $\frac{22}{24}$
6th. Add $\frac{2}{3}$ and 17 $\frac{1}{2}$ together. *Anſwer* 18 $\frac{1}{6}$
7th. Add 12 $\frac{1}{2}$ and 3 $\frac{2}{3}$ and 4 $\frac{3}{4}$ together. *Anſr.* 20 $\frac{22}{24}$
8th. Add 6 $\frac{7}{8}$ of $\frac{9}{10}$ and $\frac{4}{7}$ of $\frac{1}{2}$ and 7 $\frac{1}{2}$ together.
Anſwer 14 $\frac{1284}{2240}$
9th. Add $\frac{7}{8}$ of a pound to $\frac{3}{4}$ of a ſhilling.
Anſwer 18*s.* 3*d.*
10th. Add $\frac{3}{4}$ of a penny to $\frac{1}{9}$ of a pound.
Anſwer 2*s.* 3*d.* 1*qr.* $\frac{6}{9}$
11th. Add $\frac{1}{2}$ of a pound *Troy* to $\frac{7}{12}$ of an *oz.*
Anſwer 6 *oz.* 11 *dwt.* 16 *gr.*

12th. Add $\frac{4}{7}$ of a tun to $\frac{9}{10}$ of a *Cwt.*
Anſwer 12 *Cwt.* 1*qr.* 8*lb.* 12*oz.* 12 *drs.* $\frac{8}{10}$

13th. Add $\frac{3}{4}$ of a mile to $\frac{7}{10}$ of a furlong.
Anſwer 6 *furlongs*, 28 *poles.*

14th. Add $\frac{1}{2}$ of a yard to $\frac{2}{3}$ of a foot.
Anſwer 2 *feet*, 2 *inches.*

15th. Add $\frac{1}{3}$ of a day to $\frac{1}{2}$ of an hour.
Anſwer 8 *hrs*, 30 *min.*

16th. Add $\frac{4}{9}$ of a chaldron to $\frac{7}{8}$ of a buſhel.
Anſwer 16 *buſhels*, 3 *pecks*, $\frac{1}{2}$

17th. Add $\frac{1}{3}$ of a week, $\frac{1}{4}$ of a day, and $\frac{1}{2}$ of an hour together. *Anſwer* 2 *days*, 14 *hours*, $\frac{1}{2}$

18th. Add $\frac{2}{3}$ of a yard, $\frac{3}{4}$ of a foot, and $\frac{7}{8}$ of a mile together. *Anſwer* 1540 *yds.* 2*ft.* 9 *in.*

CHAP. III.

SUBTRACTION of VULGAR FRACTIONS.

AS the ſame rules which have already been laid down in the 1ſt, 2d, 3d and 4th ſections of the laſt chapter, for preparing Fractions for *Addition*, are to be obſerved in *Subtraction*; further inſtructions on that head are unneceſſary: when therefore they are properly prepared, this is the rule,

Rule. Subtract the leſſer *Numerator* from the greater, and the remainder placed over the *Denominator*, will be the anſwer required.

If the Fractions given to be ſubtracted are mixt numbers, and the Fraction of the mixt number to be ſubtracted, be greater than the Fraction of that is to be ſubtracted from (which you will eaſily perceive when they are reduced to a common *Denominator.*)

THEN you muſt borrow an Integer from the whole number, or numbers, to which the uppermoſt Fraction is annext (which will always conſiſt of the ſame number of parts as are expreſſed by your common *Denominator)* and ſubtracting the *Numerator* of your lowermoſt Fraction, from the Integer borrowed, or from the common *Denominator* which is the ſame thing, add the remainder to the *Numerator* of the uppermoſt Fraction, and place it below the line with the common *Denominator* under it; then carry 1 for that you borrowed, to the next figure in the lower line, and proceed as in ſubtraction of whole numbers, and you'll have the anſwer required.

Note. *This rule is ſo plain, I ſhall not ſhew the operation, as they may be proved by Addition.*

EXAMPLES.

1ſt From $\frac{96}{97}$ take $\frac{3}{4}$
Anſwer $\frac{93}{388}$.
2d From $\frac{97}{100}$ take $\frac{3}{7}$
Anſwer $\frac{307}{700}$.
3d From 48 take $\frac{3}{5}$
Anſwer 47 $\frac{2}{5}$.
4th From 24 $\frac{1}{3}$ take 15$\frac{3}{7}$.
Anſwer 8$\frac{19}{21}$.

5th From $\frac{1}{3}$ of 76 take $\frac{3}{4}$ of 21.
Anſwer 9$\frac{7}{12}$
6th From $\frac{15}{17}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$.
Anſwer $\frac{73}{102}$.
7th From 34 $\frac{3}{4}$ take $\frac{9}{10}$
Anſwer 33 $\frac{14}{40}=\frac{37}{20}$.
8th From 14 $\frac{1}{4}$ take $\frac{2}{3}$ of 19.
Anſwer 1$\frac{7}{12}$.

Note 2d. *In order to find the anſwers to the following queſtions, the remainders muſt be reduced to the known parts of an integer by the 11th ſection of the laſt chapter.*

9th From $\frac{1}{2}$ of a pound take $\frac{3}{4}$ of a ſhilling.
Anſwer 9*s.* 3*s.*

10th From $\frac{1}{4}$ of a ſhilling take $\frac{3}{4}$ of a penny.
Anſwer [illegible]*d.* $\frac{1}{4}$.

11th From $\frac{3}{5}$ of an *oz.* take $\frac{7}{8}$ of a *dwt.*
Anſwer 11 *dwts.* 3 *grs.*

12th From $\frac{1}{2}$ of an *cwt.* take $\frac{7}{12}$ of a pound.
Anſwer 1 *qr.* 27*lb.* 6*oz.* 10*dr.* $\frac{8}{12}$.

13th From $\frac{2}{3}$ of a league take $\frac{7}{10}$ of a mile.
Anſwer 1 *m.* 2 *fur.* 16 *poles.*

14th From 1 ell take $\frac{7}{10}$ of a *qr.*
Anſwer 1 *yd.* 0 *qr.* 1 *na.* $\frac{2}{10}$

15th From $\frac{3}{12}$ of a *hhd* of beer take 1 gallon.
Anſwer 12 *gall.* $\frac{1}{2}$

16th From $\frac{4}{8}$ of a chaldron take $\frac{2}{3}$ of a *buſhel.*
Anſwer 17 *buſh.* 1 *peck* $\frac{1}{3}$.

17th From 7 weeks take 9 days $\frac{7}{10}$.
Anſwer 5 *weeks,* 4 *days,* 7 *hours,* 12 *min.*

18th From 4 days, 7 hours $\frac{1}{2}$ take 1 day, 9 hours, $\frac{4}{10}$.
Anſwer 2 *days,* 22 *hrs.* ^{1}.

CHAP. IV.

MULTIPLICATION
Of VULGAR FRACTIONS.

WHEN fractions are given to be multiplied; firſt reduce them (if need be) by the rules already taught, that is, If compound Fractions, to ſimple ones; if mixt numbers, to improper Fractions; and if Fractions of different integers, to Fractions of the ſame integer,

teger; when they are thus prepared, obſerve the following

Rule. Multiply all the *Numerators* together for a new *Numerator*; and all the *Denominators* together for a new *Denominator*.

S. Muſt not they likewiſe be reduced to a common *Denominator?*

M. There is no neceſſity for it in either *Multiplication* or *Diviſion* of Vulgar Fractions.

EXAMPLES.

1ſt Multiply $\frac{2}{5}$ by $\frac{3}{10}$.
2d Multiply $\frac{5}{8}$ by $\frac{7}{9}$.
3d Multiply $\frac{2}{3}$ of $\frac{3}{5}$ by $\frac{3}{9}$ of $\frac{1}{12}$.
4th Multiply 71 $\frac{1}{4}$ by 8 $\frac{1}{2}$.
5th Multiply 4 $\frac{1}{2}$ by $\frac{1}{8}$.
6th Multiply $\frac{7}{8}$ by 13 $\frac{9}{10}$.
7th Multiply $\frac{1}{2}$ of 7 by $\frac{3}{8}$
8th Multiply $\frac{3}{8}$ of 8 by $\frac{7}{8}$ of 5.
9th Multiply $\frac{4}{5}$ of 91 by 71 $\frac{1}{2}$.
10th Multiply 12 $\frac{3}{6}$ by $\frac{2}{6}$ of 7.

11th Multiply 2*s*. 6*d*. by 2*s*. 6*d*. and let 1*s*. be the Integer. *Anſwer* 6*s*. 3*d*.

12th Multiply 2*s*. 6*d* by 2*s*. 6*d*. and let 1*l*. be the Integer. *Anſwer* 3*d* $\frac{3}{4}$.

13th Multiply 2*l*. 6*s*. 8*d*. by 2*l*. 6*s*. 8*d*. and let 1*l*. be the Integer. *Anſwer* 28*l*. 8*s* 10*d*. [illegible]

14th Multiply $\frac{1}{4}$ of a pound by $\frac{1}{4}$ of a pound.
Anſwer 1*s*. 3*d*.

S. I ſhould be obliged to you, if you will ſhew me how the 1ſt, 11th, and 12 of the foregoing examples are performed.

M. In the following manner.

Ex. 1ſt $\frac{2}{5} \times \frac{3}{10} = \frac{6}{50} = \frac{3}{25}$

Ex. 2*s.* $\frac{1}{2}$ wherefore $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4}$ of a ſhilling;

$\frac{25}{2}$ } which being reduced to its whole or mixt number according to the rule in ſection 7th, in the laſt chapter, will be found 6 $\frac{1}{4}$ of a ſhilling, which being again reduced to its proper value by the rule in ſection 11th, in the laſt chapter, will be = 6*s.* 3*d.*

Ex. 11th 2*s.* 6*d.* is $\frac{1}{8}$ of a pound, wherefore $\frac{1}{8} \times \frac{1}{8} = \frac{1}{64}$ of a pound, which being valued according to the rule in ſection 11th, in the laſt chapter, is found equal to 3*d* $\frac{3}{4}$.

S. The operations in themſelves are very ſimple, but the effects produced by them I cannot comprehend; namely (when two Fractions are multiplied into each other, as in the 1ſt and 11th examples) why the product ſhould be leſs than either of the Fractions given; or why the anſwers to the 11th and 12th, ſhould not agree, as the ſums given to be multiplied were the ſame in each, viz. 2*s.* 6*d.* by 2*s.* 6*d.*

M. In regard to your 1ſt obſervation, it may indeed ſeem ſtrange on your 1ſt conception of it; but upon a due conſideration of the following examples in whole numbers, and obſerving how their products regularly decreaſe in proportion as you leſſen the multiplier, or multiplicand, your ſurprize will then ceaſe. Let us then multiply 4 by 4.

4 } let us now leſſen the 4 } let us again leſſen the 2
4 } multiplier one half. 2 } multiplicand one half. 2
— — —
16 8 4

Let us again leſſen the 2 } and again the mul- 1
multiplier one half. 1 } tiplicand one half. 1
— —
2 1

HERE you may obſerve (in the aforegoing examples) that the 2d product is exactly $\frac{1}{2}$ of the 1ſt, the 3d, $\frac{1}{2}$ of the 2d, the 4th $\frac{1}{2}$ of the 3d, and the 5th $\frac{1}{2}$ of the 4th. Hence it muſt neceſſarily follow, that the laſt product being one, if we again leſſen the multiplier $\frac{1}{2}$, the product muſt be half of 1, viz. $\frac{1}{2}$, and if we likewiſe leſſen the multiplicand $\frac{1}{2}$ then muſt the product be $\frac{1}{4}$, and by again leſſening the multiplier $\frac{1}{2}$ the product will be $\frac{1}{8}$, and laſtly the multiplicand being leſſened $\frac{1}{2}$, the product will be $\frac{1}{16}$; ſo that is plain, multiplying proper Fractions, by proper Fractions, the product of them decreaſes in the ſame proportion, as does the product of integers when you multiply them by a part of the former multiplicand, or multiplier, as will ſtill appear more clearly by comparing the following operations, with the preceding ones in whole numbers, the laſt of which was 1 multiplied by $1 = 1$, wherefore $1 \times \frac{1}{2}$ muſt be $= \frac{1}{2}$, $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$, $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$; ſo that we here find, as was obſerved before of the whole numbers, the 2d product is $\frac{1}{2}$ the 1ſt, the 3d $\frac{1}{2}$ the 2d, the 4th $\frac{1}{2}$ the 3d, and the 5th $\frac{1}{2}$ of the 4th.

THIS clearly accounts for the operation producing ſuch effects, in all queſtions where both multiplier and multiplicand are proper Fractions, as in the firſt and eleventh example aforegoing; for in the 11th, a pound was given for the Integer, and therefore *2s. 6d.* was $\frac{1}{8}$ of that Integer, which was a proper Fraction of one pound: whereas in the 12th example (altho' the ſums given to be multiplyed were the ſame as in the 11th) yet the Integer there given was one ſhilling: wherefore *2s. 6d.* became an improper Fraction of that Integer; to which being reduced as you ſee by the operation, I then proceed to multiply them according to the firſt rule given in this chapter, and the value of the product being thus found is *6s. 3d.* as you there ſee. For Integers multiply'd by Integers, will always give for the product a greater number of the ſame Integers; whereas parts of an Integer, multiply'd by parts of the ſame Integer; muſt neceſſarily produce a ſmaller part of that Integer.

Scho. But ſuppoſe when a certain ſum of Money is given to be multiply'd by another certain ſum, we were to reduce both to one denomination, and then multiply the ſhillings, pence, or farthings of the one, by thoſe of the other, and dividing their product by 4, 12, or 20, (as in *Reduction* aſcending) would not that give the anſwer?

Maſt. No, as will appear by working the 13th example both ways.

Right way. Here 6*s.* 8*d.* being $\frac{1}{3}$ of a pound, I therefore ſet down $2\frac{1}{3}$*l.* which I reduce to an improper Fraction, thus,

$$\begin{array}{c} 2\frac{1}{3} \\ 3 \\ \hline \frac{7}{3} \times \frac{7}{3} = \frac{49}{9} \end{array}$$

which being reduced to its whole or mixt number is $5\frac{4}{9}$*l.* = 5*l.* 8*s.* $10\frac{1}{2}$*d.* $\frac{6}{9}$ or $\frac{2}{3}$ of a farthing which is the true anſwer to the queſtion.

Wrong way,

$$\begin{array}{r} £\ 2 - 6 - 8 \\ 20 \\ \hline 46 \\ 12 \\ \hline 560 \text{ pence.} \\ \times \text{ by } 560 \text{ d}^{\circ} \\ \hline 33600 \\ 2800 \\ \hline 12)313600 \\ \hline 2|0)2613|3 - 4d. \\ \hline 130l.\ 13s.\ 4d. \end{array}$$

THE difference between thoſe two anſwers are ſo very great, that I ſhould not wonder if there yet remain'd ſome doubts with you, which of the two operations are right. I ſhall therefore prove the truth of the firſt operation by Practice, thus,

£	s	d
2	6	8
by - -	2*l*. 6*s*. 8*d*.	
4	13	4
0	15	6 $\frac{1}{2}$ $\frac{2}{3}$
£5	8	10 $\frac{1}{2}$ $\frac{2}{3}$

HERE I firſt multiply the 2*l*. 6*s*. 8*d* by 2*l*, as in practical Multiplication, which gives me 4*l*. 13*s*. 4*d*: then becauſe 6*s*. 8*d*. is $\frac{1}{6}$ of 2*l*. I, therefore take the $\frac{1}{6}$ part of the product of 2*l*. 6*s*. 8*d*, multiply'd by 2*l*. which is 15*s*. 6 $\frac{1}{2}$*d*. $\frac{2}{3}$ which being added to 4*l*. 13*s*. 4*d*. gives the anſwer 5*l* 8*s* 10 $\frac{1}{2}$*d*. $\frac{2}{3}$ which is the ſame as by the firſt method of working.

HAVING now (I hope) fully prov'd the 1ſt method of working it right, I ſhall now give you ſome reaſons, why the 2d way muſt be wrong.

FIRST then you are to conſider, that the pence in 2*l*. 6*s* 8*d*. is multiply'd by the ſame number of pence; wherefore dividing it by 12, and by 20, (or which is the ſame thing by 240 which are the pence in a pound) cannot give the ſame anſwer: but if you multiply the pence in a pound, viz. 240, by 240; and by that product, divide 313600 (which is the product of the pence in 2*l*. 6*s*. 8*d*. multiply'd by the ſame number of pence) you will then have the true anſwer, as appears by the work.

```
576|00)3136|00(5l.
        2880
        ————
         256
          20
        ————
   576)5120(8s.
        4608
        ————
         512
          12
        ————
   576)6144(10d.
        576
        ————
         384
           4
        ————
  576)1536(2 far. 384/576 = 2/3
        1152
        ————
         384
```

THIS operation you ſee gives the anſwer as the 1ſt method of working did, and alſo that by Practice.

Note. The method of working this queſtion by Practice, may be very conveniently apply'd in meaſuring of Joiner's work, Plaiſterer's work, &c. as I ſhall ſhew in a chapter by itſelf, and is not only as ſhort, but the operation much more ſimple than what is commonly called croſs Multiplication.

CHAP. V.

DIVISION *of* VULGAR FRACTIONS.

WHEN the numbers given are both ſimple Fractions, multiply the *Numerator* of the Dividend by the *Denominator* of the Diviſor, the product will be the *Numerator* of the quotient: then multiply the *Denominator* of the Dividend, by the *Numerator* of the Diviſor, and that product will be the *Denominator* of the quotient.

Note. *If the Fractions given be not ſimple Fractions, they muſt firſt be reduced to ſuch, and then proceed as before directed.*

EXAMPLES.

1ſt Divide $\frac{3}{4}$ by $\frac{2}{3}$ place it thus $\frac{2}{3} | \frac{3}{4} | \frac{9}{8}$
2d Divide $\frac{2}{3}$ by $\frac{3}{4}$ do $\frac{3}{4} | \frac{2}{3} | \frac{8}{9}$
3d Divide $\frac{7}{8}$ by $\frac{9}{10}$ *Anſwer* $\frac{70}{72} = \frac{35}{36}$.
4th Divide 11 by $\frac{5}{8}$ placed thus $\frac{5}{8} | \frac{11}{1} | \frac{88}{5} = 17\frac{3}{5}$.
5th Divide $\frac{5}{8}$ by 11 do $\frac{11}{1} | \frac{5}{8} | \frac{5}{88}$
6th Divide $\frac{12}{20}$ by $\frac{3}{20}$ do $\frac{3}{20} | \frac{12}{20} | \frac{240}{60} = 4$
7th Divide $\frac{16}{80}$ by $\frac{4}{24}$ do $\frac{4}{24} | \frac{16}{80} | \frac{384}{320}. = 1\frac{1}{3}$

Note 1ſt. *The work of Diviſion in Fractions may ſometimes be very uſefully contracted in the two following caſes.*

Caſe 1ſt. When the two Fractions given, have a common *Denominator*, and the Fraction to be divided, is greater than that which you are to divide by (as in the 6th example aforegoing) where $\frac{12}{20}$ is to be to divided by $\frac{3}{20}$.

Rule. Reject the *Denominator*, and dividing 12 by 3, you have the anſwer as before. See the work.

$$3)\ 12\ (\atop \overline{\ 4\ }$$

4 *Anſwer.*

Caſe 2d. When the Fractions given are ſuch, as either the two *Numerators* or *Denominators*, can be divided ſeverally by ſome common Diviſor without a remainder (as in the 7th example aforegoing where $\frac{16}{80}$ was to be divided by $\frac{4}{24}$.

Rule. Take the Quotients inſtead of the ſaid *Numerators* or *Denominators*, and then divide according to the rule before given. So the *Numerator* 4 being divided by 4, I have $\frac{1}{24}$ for a Diviſor, and the other *Numerator* 16 being alſo divided by 4, I have $\frac{4}{80}$ for a *Dividend*. Then according to the firſt Rule the operation will be thus.

$\frac{1}{24}\Big|\frac{4}{80}\Big|\frac{96}{80}\Big|=1\frac{1}{5}$. See Example 7th.

Note 2d. *One particular advantage in working the Rule of Three by Fractions is, that we can divide a leſſer Fraction by a greater, or even a Fraction by a whole number, or numbers: whereas in performing it by whole numbers; when the two terms which are to be multiplied together, will not give a product large enough to contain the Diviſor; we are obliged to reduce the middle term into a lower* Denomination *in order to increaſe the Dividend.*

More examples in *Diviſion* to exerciſe the Learner.

8th. Divide $1\frac{1}{2}$ by $4\frac{8}{10}$. *Anſwer* $\frac{30}{96}$.
9th. Divide $\frac{7}{8}$ by 4. *Anſwer* $\frac{7}{32}$.
10th. Divide 99 by 106. *Anſwer* $\frac{99}{106}$.
11th. Divide $\frac{2}{3}$ of $\frac{7}{8}$ by $\frac{3}{4}$ of $\frac{1}{2}$. *Anſwer* $2\frac{1}{3}$.
12th. Divide $\frac{1}{6}$ of 21 by $\frac{1}{2}$ of $\frac{2}{3}$. *Anſwer* $10\frac{1}{2}$.
13th. Divide $4\frac{5}{9}$ by $\frac{5}{9}$ of 4. *Anſwer* $2\frac{1}{20}$.
14th. Divide $\frac{5}{9}$ of 4 by 4^{5}. *Anſwer* $\frac{20}{41}$.

Queſtions to exerciſe *Addition*, *Subtraction*, *Multiplication* and *Diviſion* of VULGAR FRACTIONS.

EXAMPLES.

1ſt. The difference between two numbers is $1\frac{13}{24}$ the leſſer number is $2\frac{1}{8}$ what is the greater. *Anſwer* $3\frac{2}{3}$.

2d. What number is that which if added to $12\frac{3}{8}$ will give $48\frac{2}{5}$. *Anſwer* $36\frac{1}{40}$

3d. There is in 3 bags the ſum of $420\frac{5}{8}$*l.* viz.

in the 1ſt bag 1[illegible]5 $\frac{3}{8}$l, in the 2d 135 $\frac{7}{9}$l what is in the 3d bag? *Anſwer* 59 $\frac{273}{378}$ = $\frac{13}{98}$l.

4th. What is $\frac{5}{8}$ of 250 $\frac{3}{5}$? *Anſwer* 156 $\frac{5}{8}$

5th. What number is that which being multiply'd by $\frac{3}{5}$ the product ſhall be 25 $\frac{2}{5}$. *Anſr.* 42 $\frac{1}{3}$

6th. Divide 425l. amongſt four men, let A, B and C have equal ſhares and each $\frac{1}{3}$ more than D.

Note, *Leaſt the learner ſhould miſtake this queſtion (as there are many ways by which he may work it, where the ſeveral ſhares found, being added together, would make up the whole ſum) he is therefore deſired to take notice, that* A, B *and* C *muſt each, have $\frac{1}{3}$ of* D's *ſhare more than* D. *I ſhall therefore ſhew him how it is to be performed,*

$\frac{4}{15}$)425(28 $\frac{5}{15}$ or $\frac{1}{3}$ = 6s 8d.
30

125
120

5

	£	s	d		£	s	d
	28	6	8		28	6	8
			4				3
A's ſhare	113	6	8	D's	85	0	0
B's d°	113	6	8				
C's d°	113	6	8				
D's d°	85	0	0				
Proof	425	0	0				

Scho. I dont underſtand the reaſon of this operation.

Maſt. I ſhall explain it to you: Firſt then I conſider that if the whole ſum were divided into 5 equal parts; that A, B, C, and D, muſt ſhare equally in 4 of theſe parts; and the 5th part being again divided equally amongſt A, B and C, would give each $\frac{1}{3}$ of $\frac{1}{5}$ more, which being reduced to a ſimple Fraction is $\frac{1}{15}$ of the whole; If therefore A, B and C have each $\frac{1}{15}$ of the whole more than D, conſequently they muſt have $\frac{3}{15}$ of the whole more amongſt them, wherefore I ſubtract $\frac{3}{15}$ from $\frac{1}{1}$ or $\frac{15}{15}$, and the remainder is $\frac{12}{15}$, which is to be equally divided amongſt all the four, thus, $\frac{4}{1})\frac{12}{15}(\frac{12}{60} = \frac{3}{15}$ to which if we add $\frac{1}{15}$ is $\frac{4}{15}$ the ſhare that A, B and C are each to have of the whole; wherefore having placed 425*l*. for my dividend and $\frac{4}{15}$ for my diviſor as you ſee by the operation, I divide 425*l*. by 15, which gives the quotient, 28*l*. 6*s*. 8*d* which is $\frac{1}{15}$ of the whole. Then becauſe A, B and C are each to have $\frac{4}{15}$ I therefore multiply that ſum by 4, and the product is 113*l*. 6*s*. 8*d*. the reſpective ſhare of A, B and C. Now if each of them have $\frac{4}{15}$ they have amongſt them $\frac{12}{15}$, the remaining part viz. $\frac{3}{15}$ muſt therefore of courſe be the ſhare of D; wherefore multiplying again 28*l*. 6*s*. 8*d*. (viz $\frac{1}{15}$) by 3, gives the whole ſhare of D, namely 85*l*: ſo that A, B and C have each 85*l*. and $\frac{1}{3}$ of 85*l*. to their ſhare, as may be proved thus,

$$
\begin{array}{lrrr}
3)85(& & & \\
& \pounds & s & d \\
28\frac{1}{3} = & 28 & 6 & 8 \\
& 85 & 0 & 0 \\
\hline
& 113 & 6 & 8
\end{array}
$$

CHAP. VI.

Of the SINGLE *Rule of* THREE DIRECT *and* INVERSE, *in* Vulgar Fractions.

AS the manner of ſtating and working queſtions in the Rule of Three in Fractions (either Single or Double) is exaƈtly the ſame as in whole nur reſpeƈt being had to Multiplication and Divil Fraƈtions, and alſo to the manner of reducing the would therefore be ſuperflous to repeat theſe i tions over again. I ſhall therefore content myſe giving you a few examples to ſhew you in what queſtions may be more conveniently performed by than by whole numbers; by which you will eaſil when to apply them with advantage to any rules already taught.

If 12 yards of cloth coſt 3*l*. 6*s*. 8*d*. what wi coſt?

$$\begin{array}{lcccccc} & \textit{yds} & & \pounds & & \textit{yds} \\ \text{If} & \frac{12}{1} & : & \frac{10}{3} & :: & \frac{123}{4} \end{array}$$

$$\frac{12}{1})\ \frac{1230}{12}\ (\frac{1230}{144} \qquad 144)\ 1230\ (8\ \frac{78}{144} =$$

$$\begin{array}{r} 1152 \\ \hline 78 \end{array}$$

SO that I have for the anſwer 8 $\frac{13}{24}$*l*. whic valued will be equal to 8*l*. 10*s*. 10*d*.

S. I don't underſtand this operation.

M. Firſt I reduce all the terms to imprope tions, and then ſtate the queſtion as you ſee; multiply my 2d and 3d terms together, viz. 12 for a *Numerator*, and 4 by 3 for a *Denominator* gives me $\frac{1230}{12}$, which being divided by $\frac{12}{1}$ I ha for the anſwer. I then reduce it it to its whole number which gives me 8 $\frac{78}{144}$*l*. or in its lowe 8 $\frac{13}{24}$*l*. which Fraƈtion being reduced to its prop is 10*s*. 10*d*.; ſo that I find the whole anſwe 8*l*. 10*s*. 10*d*.

BY this queſtion you may eaſily perceive the tage of working fraƈtionally; for if you were the ſame by whole numbers, you muſt have reduced your middle term to pence, and your laſt term into quarters, and conſequently your firſt terms into quarters alſo. Then after you have multiplied your 2d and 3d terms together, and divided the product by the 1ſt term, the anſwer which would have been pence, muſt again have been divided by 12 and 20, which would have been a very tedious operation.

2d If $\frac{7}{8}$ of a yard of cloth coſt $\frac{3}{4}$ of a pound, what will $\frac{2}{3}$ of a yard coſt?

Anſwer $\frac{48}{84}$ of a pound = 11*s.* 5*d.* $\frac{1}{7}$.

3d If 6$\frac{1}{2}$ yards of cloth, 2 yards wide, will cover a table, how many yards will cover the ſame table when the cloth is half a yard wide.

Anſwer $\frac{52}{2}$ = 26 *yards.*

4th If 12 men mow 24$\frac{1}{2}$ acres in 10$\frac{2}{3}$ days, in what time will 3 men mow the ſame?

Anſwer 42 $\frac{2}{3}$ *days*

5th If 3 men can mow 24 $\frac{1}{2}$ acres, in 42 $\frac{2}{3}$ days in what time will 12 men do the ſame?

Anſwer in 10 $\frac{2}{3}$ *days.*

6th If $\frac{11}{12}$ of a *cwt.* coſt 1*l.* 13*s.* 4*d.* what will 3 $\frac{1}{2}$ *cwt* coſt.

Anſwer 6 $\frac{4}{11}$*l.* = 6*l.* 7*s.* 3*d.* $\frac{1}{4}$ $\frac{1}{11}$.

7th If a ſilver ſalt weighing 3 *oz.* coſt 15*s.* 9*d.* what will a bowl coſt that weighs 2 $\frac{1}{3}$ *lb.*

Anſwer 163 $\frac{2}{3}$*s.* = 8*l.* 3*s.* 9*d.*

8th If I lend my friend 50*l.* for 7 months, allowing him 4 weeks to the month, how much ought he lend me for 3 weeks to requite the favour?

Anſwer 466 $\frac{2}{3}$*l.* = 466*l*, 13*s.* 4*d.*

Note. *When the* Numerator *of the 2d term, and the*

Denominator *of the* 3d *term, can be ſeverally divided by any one number; and the* Numerator *of the* 3d, *and* Denominator *of the* 2d *term, by the ſame, or any other number; your work may be greatly contracted, as will appear by the following queſtion.*

9th If $\frac{3}{4}$ of a yard of velvet, coſt 7*s.* 3*d.* what will 7 $\frac{2}{3}$ yards coſt? Your numbers being reduced and ſtated as afore taught, your work will ſtand thus,

$$\begin{array}{cccccc} & \textit{yd} & & \textit{£} & & \textit{yd} \\ \text{If} & \frac{3}{4} & : & \frac{87}{240} & :: & \frac{30}{3} \end{array}$$

HERE you may obſerve that the *Numerator* of the laſt term, and the *Denominator* of the 2d may each of them be divided by 5; and again the *Numerator* of the 2d term, and the *Denominator* of the laſt, may each be divided by 3, and the queſtion will be contracted thus.

$$\begin{array}{cccccc} & \textit{yds.} & & \textit{£} & & \textit{yds.} \\ \text{If} & \frac{3}{4} & : & \frac{29}{48} & :: & \frac{6}{1} \end{array}$$

S. You have before fully proved to me, that the *Numerator* and *Denominator* of any 1 Fraction, being multiplied or divided by 1 and the ſame number, that the new Fraction ſo produced will be equal to the 1ſt Fraction given; how then can the *Numerator* and *Denominator* of a Fraction divided by 2 different numbers (as they are in this caſe) produce a Fraction of equal value with the 1ſt given Fraction?

M. Neither of the new Fractions ſo produced, will be equal in the value to the 1ſt given Fractions, nor will they bear the ſame proportion one to the other; but when two Fractions are to be multiplied together, then in that caſe, the product of the *Numerators* of the two new Fractions, ſhall bear the ſame proportion to the product of their *Denominators*, as the product of the 1ſt *Numerators*, did to the product of their *Denominators*. For if one of the *Numerators* of the 1ſt given Fraction be divided by 5, and the *Numerators* of the other by 3, as in the laſt queſtion; then muſt it be the very ſame thing as if both *Numerators* had been multiplied into each other, and the product divided by 15, 5 times 3 being 15, and the like of the *Denominators*; of conſequence then, the *Numerators* and *Denominators* being each 15 times leſs, if we divide one by the other it will give the ſame quotient, as would the product of the 1ſt *Denominators*, divided by the product of their *Numerators*. For a proof of this

Let us therefore multiply the two Numerators of the 2d and 3d terms in the laſt queſtion together, and likewiſe their Denominators, thus,

	87		240
	30		3
Product of the Numerators --	2610	Product of the Denominators --	720

Now if I would have both the above products three times leſs, then I muſt make each of their multipliers, or multiplicands (no matter which) three times leſs. If I would yet have thoſe products 5 times leſs; 'tis only making the multiplicand, or multiplier of each, again 5 times leſs; and their products will ſtill have the ſame proportion to each other, as the products of the firſt Fractions given had: for any two numbers given to be multiplied together, 'tis no matter which you make your multiplier or multiplicand. Suppoſe then the two laſt examples were placed in this manner,

87	3
30	240
2610	720

Now becauſe I would have one of the given terms of each, ſuch a number, as to make the product three times leſs; I therefore divide 87 and 3, each by 3, which gives me 29 and 1: but I would ſtill have the product five times leſs: therefore, I again divide 30 and 240, by 5, which gives me, as you ſee - - 6 & 48

So that having multiplied 1 by 48, and 29 by 6, I have 174 & 48 theſe for the two products: which two numbers bear *the ſame proportion* to each other: as 720 did to 2160, as will appear by dividing the greater of each by the leſſer.

$$\begin{array}{ll} 48)174(3\ \frac{30}{48}=\frac{5}{8} & \qquad 720)2610(3\ \frac{450}{720}=\frac{5}{8} \\ \quad 144 & \qquad \quad 2160 \\ \quad \overline{} & \qquad \quad \overline{} \\ \quad 30 & \qquad \quad 450 \end{array}$$

S. I ſee it now very plain.

M. You may likewiſe often further abbreviate your work, by dividing the *Denominators* of your 1ſt and 2d terms (if they will admit of it) by ſome common diviſor; and altho' in this caſe, the Fractions ſo abbreviated will not (if conſidered ſeparately) retain their reſpective values; yet will they have the ſame proportion to each other as before. For a proof of this, let us again ſtate the ſame queſtion as it ſtood abbreviated by the firſt contraction, which was
thus, *Yd* £ *yd* and here having divi-
If $\frac{3}{4} : \frac{20}{48} :: \frac{6}{1}$ ded the *Denominators*
of my 1ſt and 2d terms, by 4, the queſtion will then ſtand thus, If $\frac{3}{1} : \frac{20}{12} :: \frac{6}{1}$ Now it is plain, that the Numerator of the Dividend being multiplied by a number 4 times leſs, which (according to the manner of dividing Fractions) is by the *Denominator* of your 1ſt term; and the *Denominator* of your dividend (which is to be multiplied by the *Numerator* of the diviſor) being likewiſe 4 times leſs; conſequently, the *Numerator* and *Denominator* of your diviſor muſt be leſſened exactly in the proportion.

S. If we were to divide the *Numerator* and *Denominator* of the 2d term, by 1 one number, and the *Numerator* and *Denominator* of the 3d term by another number; would the proportion be ſtill the ſame?

M. No, each Fraction ſeparately would retain the ſame value, but the two Fractions could not bear the ſame proportion one to the other, as they did before; being divided by two different numbers; but if you could find a number which would divide both *Numerators* and both *Denominators*, then would they have the ſame proportion to each other, as the firſt given Fractions.

S. Or ſuppoſe we could not find two numbers which would divide each of the *Numerators* and *Denominators* of the 2d and 3d terms alternately (as in the 1ſt contraction, would dividing any of the *Numerators* and *Denominators* of the other Fraction, by ſome common diviſor, give the ſame anſwer.

M. Yes, which I ſhall prove to you, by working the afore going queſtion, according to all the different contractions. As it was 1ſt propoſed thus.

$$\text{If } \tfrac{3}{4} : \tfrac{87}{240} :: \tfrac{30}{1}$$

$$\tfrac{3}{4}\Big|\ \tfrac{2610}{720}\ \Big|\ \tfrac{10440}{2160}\ \textit{Anſ.} \qquad \begin{array}{rr} 87 & 240 \\ 30 & 3 \\ \hline \textit{Num. } 2610 & \textit{Denom. } 720 \end{array}$$

SO that I have $\frac{10440}{2160}$ which being reduced to its whole or mixt number thus,

$$\begin{array}{r} 2160)\ 10440\ (4 \\ 8640 \\ \hline 1800 \end{array}$$

I have 4*l.* $\frac{1800}{2160}$ or $\frac{5}{6}$ which being reduced to its proper value is = 4*l.* 16*s.* 8*d.*

LET us now try, if it will give the ſame anſwer, when the *Numerator* of the 3d term, and *Denominator* of the 2d, are abreviated by dividing each of them by 5, when the terms would ſtand ranged in this manner.

Contraction 1ſt If $\frac{3}{4} : \frac{87}{48} :: \frac{6}{1}$

$$\left.\begin{array}{l} \text{here } 87 \times 6 = 522 \\ \text{and } 48 \times 3 = 144 \end{array}\right\} \frac{522}{144} \div \frac{3}{4} = \frac{2088}{432} = 4\ \frac{5}{6}$$

of a pound as before.

LET us again abreviate the *Numerator* of the 2d, and *Denominator* of the 3d term by dividing each by $\frac{3}{4}$, when the work will ſtand thus.

Contraction 2d If $\frac{3}{4}$: $\frac{29}{48}$ :: $\frac{6}{1}$

$$\begin{array}{l} \text{here } 29 \times 6 = 174 \\ \qquad\qquad\qquad\qquad \overline{} \div \frac{3}{4} = \frac{696}{144} = 4\frac{5}{6}\,l. \\ \text{and } 48 \times 1 = \ \ 48 \end{array}$$

And laſtly, Let us abreviate the work again, by dividing the Denominators of the 1ſt and 2d terms in the laſt ſtating, by dividing each of them by 4, when the queſtion would be ſtated thus,

$$\begin{array}{lccc} & Yd & £ & yd \\ \textit{Contraction } 3d. \ \text{If} & \frac{3}{1} : & \frac{29}{12} : : & \frac{6}{1} \end{array}$$

$$\begin{array}{l} \text{Here } 29 \times 6 = 174 \\ \qquad\qquad\qquad\qquad \overline{} \div \frac{3}{1} = \frac{174}{36} = 4\frac{5}{6}\,l. \\ \text{and } 12 \times 1 = 12 \end{array}$$

Note. *Inſtead of making 7s. 3d. (viz the price of $\frac{3}{4}$ yard) the Fraction of a pound, you might if you pleaſe, have made it the improper Fraction of a ſhilling, and then the anſwer would have been the improper Fraction of a ſhilling; whereas by making it a Fraction of a pound, we have an improper Fraction of a pound.*

CHAP. VII.

Of the DOUBLE RULE of THREE, in VULGAR FRACTIONS,

Perform'd by one ſingle Stating; or by two ſingle *Rules* of *Three*.

THE manner of ſtating and working the Double *Rule* of *Three* in Fractions, is in all reſpects the ſame as in whole numbers, regard being had (as has been obſerved before) to the manner of preparing your terms by Reduction, and likewiſe to Multiplication and Diviſion of Fractions.

EXAMPLES.

1ſt. If 50 pounds in nine months gain 2*l.* 13*s.* 4*d.* intereſt; what intereſt will 260*l.* 3*s.* 4*d.* gain in 5 months?

$$\begin{array}{cc} \pounds & \pounds \\ 260\frac{1}{6} & 2\frac{2}{3} \\ 6 & 3 \\ \hline 1561 & 8 \\ \hline 6 & 3 \end{array}$$

If $\frac{50}{1}$*l.* : $\frac{9}{1}$*m.* : $\frac{8}{3}$*in.* :: $\frac{1561}{6}$*l.* : $\frac{3}{1}$*m.*

$\frac{3}{1} \times \frac{1561}{6} = \frac{4683}{6} \times \frac{8}{3} = \frac{37464}{18}$ for a Dividend.

$\frac{50}{1} \times \frac{9}{1} = \frac{450}{1}$ for a Diviſor, $\frac{450}{1})\frac{37464}{18}(\frac{37464}{8100}$

$8100)37464(4\frac{5064}{8100} = 4$*l.* 12*s.* 6*d.* $\frac{30}{675}$

$$\begin{array}{r} 32400 \\ \hline .5064 \end{array}$$

HAVING firſt reduced my 260*l.* 3*s.* 4*d.* and 2*l.* 13*s.* 4*d.* to the improper Fraction of a pound, I next proceed to ſtate my queſtion as directed in the double *Rule* of *Three* compounded of five numbers; and finding it to be direct, I next begin to work it according to the rule (in that caſe) given, viz. I firſt multiply my 4th and 5th terms together, and that product again by my 3d or middle term, for a Dividend; I next multiply my 1ſt and 2d terms together for a Diviſor, and dividing the one by the other the quotient is $\frac{37464}{8100}$ of a pound; and laſtly becauſe it is an improper Fraction, I divide the *Numerator* by the *Denominator* and I have 4*l.* and $\frac{5064}{8100}$ of a pound which is equal to 12*s.* $6\frac{30}{675}$*d.*

2d. If 12 men ſpend 15*l* $\frac{7}{9}$ in 18 days, how much will 25 men ſpend in 40 days?

Anſwer 73*l*. 0*s*. 10*d*. $\frac{3}{4}$ $\frac{898}{1944}$.

3d. If 8 men in 7 days earn 9 $\frac{2}{6}$ pounds, what will 56 men earn in 21 $\frac{1}{2}$ days?

Anſwer 200$\frac{2}{3}$ *pounds*.

4th. If the wages of 12 men for a week be 11*l*. 11*s*. what number of men may be imploy'd one year (or 52 weeks) for 5405 $\frac{2}{5}$ pounds?

Anſwer 108 *men*.

HAVING now I hope ſufficiently prov'd to you (by thoſe few examples, and thoſe in the ſingle rule of Three) that many queſtions in common uſe, may be much more conveniently performed by Fractions, than by whole numbers; I ſhall next ſhew you, their particular uſe and excellence, in applying them to queſtions, ſome of which cannot be ſolved by whole numbers, and which are generally perform'd by Algebra.

CHAP. VIII.

A Collection of pleaſant and ſubtil Queſtions to exerciſe the learner in Vulgar Arithmetic, *which will more particularly ſhew the uſe and excellence of* Vulgar Fractions.

1ſt. A certain gentleman of the turf laid 9 bets, and loſt them all. He laid $\frac{1}{4}$ $\frac{1}{5}$ $\frac{1}{6}$ $\frac{1}{8}$ $\frac{1}{12}$ and $\frac{1}{20}$ of the ſum he had, at different bets: he loſt 20*l*. to one, and 30*l*. to another, and then he had 30*l*. left; I demand the whole ſum he had when he firſt began to bet, and what he loſt to each perſon he betted with.

To perform this queſtion, you are to take notice, that the gentleman laid 9 bets: but $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, & $\frac{1}{20}$ amount but to ſix of them. Therefore the 20 pounds which he loſt with one, and the 30 pounds which he loſt with another, muſt be two bets more, which make 8; and then we are told he had 30 pounds left. which 30 pounds he muſt likewiſe have betted and loſt, to make out the 9 bets. I therefore reduce $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{8}$, $\frac{1}{12}$, & $\frac{1}{20}$ to a common Denominator, by the rule given in the 2d Section of the 1ſt Chapter; and then adding them together, by the 1ſt rule in the 2d Chapter, I find the ſum of them to be $\frac{20160}{23040}$, which being reduced to its loweſt terms, by the 1ſt Section of the 1ſt Chapter will be found $= \frac{7}{8}$. If then $\frac{7}{8}$ of his whole ſum, be equal to the ſum of all thoſe bets, then muſt the difference between $\frac{7}{8}$ and his whole ſum (which is $\frac{1}{8}$) be equal to the two 30 pounds, and the 20 pounds aforementioned. Wherefore I ſay,

If $\frac{1}{8} : \frac{80}{1} :: \frac{1}{1}$

which being wrought according to the rule of Three in Fractions, the anſwer will be 640*l*. which was his whole ſtock when he began to bett. And laſtly, dividing 640*l*. by 4, 5, 6, 8, 12, & 20, we have the ſums of thoſe ſeveral betts, as you ſee by the work.

4) 640 (5) 640 (6) 640 (

  160   128   106(= or 13*s* 4*d*.

8)640(12)640(

  80   53 = =6*s* 8*d*.   2|0)64|0

	£ *s* *d*
	160- 0-0
	128- 0-0
	106-13-4
Which being placed in this manner are the ſeveral ſums = , , , , & .	80- 0-0
	53- 6-8
	32- 0-0
To which if we add the two 30 pounds	30- 0 0
and the 20 pounds which he loſt beſides,	30- 0-0
Thus	20- 0 0
Gives us the whole he had,	640- 0 0
And fully proves the work.	

2d. A man dying gave to his eldeſt ſon $\frac{2}{3}$ of $\frac{3}{4}$ of his eſtate, to his ſecond ſon $\frac{1}{5}$ of $\frac{1}{2}$ of his eſtate, and when they had counted their portions, the one had 40*l* more than the other: the remainder of the eſtate was given to the wife and younger children. I demand what was the portion of the eldeſt ſon, what the portion of the ſecond, and how much belonged to the wife and younger children?

To ſolve this queſtion, I firſt reduce the eldeſt ſon's ſhare, viz. $\frac{2}{3}$ of $\frac{3}{4}$ to a ſimple fraction, which I find is $\frac{6}{12} = \frac{1}{2}$, and in like manner reduce the ſecond ſon's ſhare to a ſimple fraction, which is $\frac{1}{10}$ .Now becauſe we are told in the queſtion, that the one had 40*l* more than the other, I therefore reduce $\frac{1}{2}$ and $\frac{1}{10}$ to a common Denominator, and ſubtracting the one from the other, I find the difference to be $\frac{1}{15}$. Wherefore 40*l* muſt be equal to $\frac{1}{15}$: I therefore, by the rule of Three ſay, If $\frac{1}{15} : \frac{40}{1}l :: 1$
which is the whole Eſtate, and which, by the operation, will be found 600*l*.

Then again I ſay, If $\frac{1}{15} : \frac{40}{1}l :: \frac{1}{6}$ which is the eldeſt ſon's ſhare: this by the operation will be found 100*l*.

	£
If then the eldeſt ſon's portion be - -	100
Conſequently the ſecond ſon's (being 40 leſs) muſt be - - - -	60
The ſum of which two being ſubtracted from the whole Eſtate, viz. 600*l*, the remainder muſt be - -	440
Which being all added, amounts to the whole Eſtate - - - - - -	600

The wife & ygr. chidrens ſhare.

3d A young man received 66 $\frac{2}{3}$ *l*. which was $\frac{2}{3}$ of $\frac{1}{2}$ of his eldeſt brother's portion, and 2 $\frac{1}{2}$ times his eldeſt brother's portion was 1 $\frac{2}{3}$ times the father's eſtate, the queſtion is what was the father's eſtate?

HERE having firſt reduced 66 $\frac{2}{3}$ *l.* to the improper Fraction $\frac{200}{3}$ alſo the $\frac{2}{3}$ of $\frac{1}{2}$ to the ſimple Fraction $\frac{2}{6} = \frac{1}{3}$; I then proceed to find out what was the eldeſt brother's portion in this manner ſaying, If $\frac{1}{3}$: $\frac{200}{3}$ :: $\frac{1}{1}$ which by the operation will be found 200*l.* which 200*l.* × 3 $\frac{1}{2}$ or $\frac{7}{2}$ = 700; then by the Rule of Three, I ſay again If 1 $\frac{1}{4}$: $\frac{700}{1}$: : $\frac{1}{1}$ and find the anſwer 560*l.*

4th A brazen lion in an artificial fountain, convey'd water into a ciſtern by 2 ſtreams iſſuing from his eyes, alſo 1 from his mouth. and by another at the bottom of his right foot. Now the pipes through which theſe ſtreams paſs, are of different capacities, in ſuch ſort, that by the right eye ſet open alone, the reſt of the ſtreams being ſtopt, the ciſtern will be filled in 2 days (the length of a day being ſuppoſed to be 12 hours) by the left eye alone, in 3 days; by the foot alone, in 4 days; and by the mouth alone, in 6 hours. The queſtion is, to find in what time the ciſtern will be filled, if all theſe ſtreams be ſet open at once?

Anſwer $\frac{12}{37}$.

FIRST I find how many ciſterns would be filled, if they were all to be ſet open at once for any certain time, (as for inſtance 3 days, which is the time in which the left eye would have filled 1 ciſtern) in this manner,

	days		*ciſt.*		*days*		*ciſtern*
If the right eye in	2	:	1	: :	3	*Anſwer*	1 $\frac{1}{2}$.
If the foot in	4	:	1	: :	3	d°	4 $\frac{3}{4}$
If the mouth in	$\frac{1}{2}$	:	1	: :	3	d°	6
6 hours or	then becauſe the left foot filled it in 3 days we therefore add.						1 more.
						total ſum	9 $\frac{1}{4}$.

HENCE it appears that the 9 $\frac{1}{4}$ ciſterns would be filled in 3 days, by all the four ſtreams running together, I therefore by the rule of Three ſay,

ciſt *days* *ciſt*
If 9 $\frac{1}{4}$: $\frac{3}{1}$ :: $\frac{1}{1}$ which being wrought according to the Rule of Three in Fractions, gives me the anſwer $\frac{12}{37}$ days.

5th A ciſtern in a certain conduit is ſupplyed with water by a pipe of ſuch bigneſs, that if the cock A at the end of the pipe be open, the ciſtern will be filled in $\frac{1}{2}$ hour; there are likewiſe at the bottom of the ciſtern two cocks B and C, whoſe capacities are ſuch, that by the cock B ſet open alone (all the reſt being ſtopt, the ciſtern ſuppoſed to be full will be emptied in 1 $\frac{3}{7}$ hour; alſo by the cock C ſet open alone, the ciſtern will be emptied in 2 $\frac{4}{5}$ hours. Now becauſe more water will be infuſed by the cock A, than can be expelled by both the cocks B and C in one and the ſame time; the queſtion is; to find in what time the ciſtern will be filled, if all the ſaid 3 cocks be ſet open at once? *Anſwer* 1. $\frac{9}{31}$ *hours*.

FIRST (according to the manner of the laſt queſtion) find how many times the ciſtern would be emptied in one and the ſame ſpace of time, by the cocks B and C running together; alſo find how much of the ciſtern will be filled by A in the ſame time (ſuppoſing the other two to be ſtopt, and the ciſtern full) then will the difference ſhew how much of the Ciſtern is gained by the filling cock A in the ſame time: and laſtly by the Rule of Three, ſay, As the ſaid difference is to the time in which it was gained; ſo will 1 ciſtern be to the time in which it will be filled when all three are running together, which gives the anſwer 1 $\frac{9}{31}$ *hours*.

6th. Suppoſe a Dog, a Wolf, and a Lion were to devour a Sheep, and that the Dog could eat up the ſheep in an hour, the wolf in $\frac{3}{4}$ hour, and the lion in $\frac{1}{2}$ hour: now if the lion begin to eat $\frac{1}{8}$ hour before the other two, and afterwards all three eat together; the queſtion is in what time the ſheep would be devoured?

Anſwer $\frac{11}{104}$ *hour.*

After the manner of the laſt queſtion, find what each would eat in one and the ſame time (for inſtance $\frac{1}{2}$ hour the time in which the lion would have devoured, it) which added all together will be the quantity which they would all have eat in $\frac{1}{2}$ hour: then find how much the lion eat in $\frac{1}{2}$ hour (which was the time he eat before the reſt began) next ſubtract that from the whole ſheep, and the difference will be what remain d of the ſheep to be eaten: then ſay, as the whole quantity eat by them all in $\frac{1}{2}$ hour, is to $\frac{1}{2}$ hour, ſo will the remainder of the ſheep be to the time in which they would all eat it: and laſtly adding the $\frac{1}{2}$ to the time ſo found, you have the anſwer $\frac{1}{104}$1 hour.

The END *of* VULGAR FRACTIONS.

◆

OF

OF
Decimal Fractions.

CHAP. I.

DECIMALS are a kind of artificial Fractions, where the Denominator is never expreſt, but underſtood; and is always an Unit, with as many Cyphers annext to it, as there are places in your Numerator: ſo .5 would be $\frac{5}{10}$; .25, $\frac{25}{100}$; and .225, $\frac{225}{1000}$, &c.

IN Decimals we ſuppoſe an Integer (of whatever ſort or kind) to be divided into ten equal parts, and each of thoſe parts, into ten other equal parts, and ſo on without end.

A *Decimal* is diſtinguiſhed from an *Integer* by a point, or comma placed before it thus .5 or thus ,5

NOTATION or NUMERATION. OF DECIMALS.

THE value of *Decimal* places increaſes towards the left hand, and decreaſe towards the right in the ſame proportion that Integers do, as will appear by the following table.

TABLE

TABLE.

Hundred Thouſands	*Ten Thouſands*	*Thouſands*	*Hundreds*	*Tens*	*Units*	*Tenth parts*	*Hundred parts*	*Thouſandth parts*	*Ten Thouſandth parts*	*Hundred thouſandth parts*
6	5	4	3	2	1	,1	2	3	4	5

HERE you ſee that every unit in the figure towards the left, is ten times the value of an unit in that figure which ſtands next it on the right hand for 100000 parts, is equal to one 10000 part, ten 10000 parts = one 1000 part, ten 1000 parts = 100 part, ten 100 parts = one 10th part, one 10th part = 1 unit, 10 units = 1 ten, ten 10 = 100, &c. &c. &c.

CHAP. II.

Of ADDITION and SUBTRACTION of DECIMALS.

IN adding or ſubtracting *Decimals*, care muſt be taken to place the numbers regularly under each other according to their ſeveral values, viz. in Integers the firſt figures of each line to the right hand (which is the place of units) under each other, tens under tens, &c. and in the Decimal parts, the firſt figure in each line towards the left hand under, each other ſeparating the

Integers, from the Decimal parts, by a point or comma as before, which muſt likewiſe be regularly placed in perpendicular line one under another: when your numbers are thus placed, proceed to add or ſubtract them the ſame as whole numbers, dividing the Integers from the Decimal parts, (in the ſum or difference) by a point, placed directly under the others.

Examples in ADDITION.

Shillings.	*yards.*	*£*	*miles.*
25 210	47 4	27.5	41 812
1.3250	18.57	5.6	150.72
5.27	271.164	52.543	18 1
322.10	7.426	75.6	24 350
4.27	8 25	4.24	64.62
5.64	25.3	5.72	7 2
363.8150			

acres.	*ounces.*
.72364	56.4230
.8542	2.6321
.012	3.2752
.87	.7013
.04	.0212
.4	.0238
2.89984	

Examples in SUBTRACTION.

	years	*days*	*weeks*	*hours*
From	231 724	721.20008	217 18	12.
Take	106.635	8.342	124.	.12
	125 089			

From

From 121.1	7400.	.174621	426.816
Take 1.264	6.209	.006437	28.945

Note, *The reaſon why you carry at tens in* Addition *and* Subtraction *of* Decimals, *is the ſame with that in whole numbers; every figure from the firſt, towards your right hand in your Decimal places, increaſing in a ten fold proportion to the figure which ſtands furtheſt to the left hand in your Integer: for each figure next the point in your Decimal places having* 10 *for its Denominator, conſequently ſo many tens as are in the ſum of that line, muſt be ſo many units in the next line, which is the firſt place of Integers.*

CHAP. III.

MULTIPLICATION of DECIMALS.

MULTIPLICATION of Decimals, both in placing your figures, and in the work, is the ſame as in whole numbers; only when your work is finiſhed you muſt remember to point off as many Decimal places to the right hand as there are in both your Multiplicand and Multiplier; but in caſe there be not ſo many figures in your product, you muſt prefix cyphers to the left hand of it, and then place your point before them.

Multiply .425	Mult. .721	Mult. 7.615
By .75	by 1.75	2.15
2125	3605	38075
2975	5047	7615
.31875	721	15230
	1.26175	16.37225

4th Multiply 48 by 48.
5th Multiply 1.83 by 72.
6th Multiply .00071 by .123.
7th Multiply .0041 by 41.
8th Multiply 37.201 by .54.
9th Multiply 460.3 by 54.
10th Multiply 27.04 by .005.
11th Multiply .0005 by .004.

S. Why muſt there be as many Decimal places in the product, as there are Decimal places in both Multiplicand and Multiplier?

M. To make this plain to you, I muſt deſire you will mark well the following obſervations.

1ſt As I told you in the introduction to Decimals; the *Denominator* of any Decimal Fraction conſiſts always of an unit with ſo many cyphers annext to it, as there are places, in the *Numerator;* conſequently then, it muſt either be 10, 100, 1000, 10000, &c. ſtill increaſing in a ten fold proportion.

2d *Every figure in the* Numerator, *has its particular* Denominator; *viz. the* 1ſt *figure towards the left hand, next the point, has* 10 *for its* Denominator; *the* 2d *figure* 100, *the* 3d *figure* 1000, *&c.* So in this Decimal .625 the 1ſt figure to the left hand is $\frac{6}{10}$, the 2d gure is $\frac{2}{100}$, and the 3d figure is $\frac{5}{1000}$. which being all reduced to a common *Denominator* and added together, will be found $= \frac{625}{1000}$.

THOSE things being premiſed, I ſhall endeavour to prove to you, from Multiplication of Vulgar Fractions, the reaſon why (in Multiplication of Decimals) we muſt have as many Decimal places in the product as there are in both the Multiplicand and Multiplier.

FIRST then according to Multiplication of Vulgar Fractions, let us multiply $\frac{2}{5}$ by $\frac{5}{8}$ which will be $\frac{10}{40} = \frac{1}{4}$.

NOW let us multiply the ſame decimally; but 1ſt (becauſe we muſt have the *Denominators* 10, 100, 1000,

&c.

&c.) we muſt find the two *Numerators* which will bear the ſame proportion to ſome of thoſe, as 2 does to 5, and 5 to 8; wherefore according to the rule in the 3d ſection in Reduction of Fractions, I ſay,

d *n* *d* *d* *n* *d*
If 5 : 2 : : 10 and again If 8 : 5 : : 1000

and find the 1ſt gives me 4 for a *Numerator*, inſtead of 2, which has 10 for its *Denominator* $=\frac{4}{10}$; and for the 2d, I have 624 for a *Numerator* inſtead of 5, which has 1000 for its *Denominator* $=\frac{625}{1000}$; wherefore cancelling the two *Denominators*, I have decimals which repreſent the two fractions, viz. .4 and .625 whoſe *Denominators* are underſtood: for the firſt Decimal conſiſting only of 1 place, muſt have an unit and a a nought for a *Denominator*, and the 2d conſiſting of of 2 places, muſt have an unit and 2 noughts for its *Denominator*, as I obſerved in the introduction to Decimals, and which obſervation you ſee agrees with the above method of finding them. Now let us multiply the Decimal .4 and .625 together, and ſee if the product agrees with the product of $\frac{4}{10}$ multiplied by $\frac{625}{1000}$ fractionally; firſt then,

.625
by .4
———
.2500 here you ſee I have the Decimal .2500 for the product, whoſe *Denominator* according to the rule given is underſtood to be 1 unit, with four noughts annext to it, or 1000.

Let us now again multiply $\frac{4}{10}$ by $\frac{625}{1000}$ fractionally, $\frac{2500}{10000}$ this you ſee exactly agrees with the Decimal operation, and being reduced to its loweſt terms will be equal to the product of the two firſt Fractions given, viz $\frac{2}{5}\times\frac{5}{8}=\frac{10}{40}$ or $\frac{1}{4}$.

S. But why as you told me in the introduction, when there are not ſo many figures in the product as there are Decimal places in both Multiplicand and

Multiplier, ſhould we prefix cyphers to make it equal to that number?

M. For this reaſon, let us ſuppoſe for inſtance $\frac{2}{10}$ are to be multiplied by $\frac{4}{10}$; then according to Fractions, the product would be $\frac{8}{100}$, now let us multiply it decimally,
.4 here you ſee the product conſiſting only of
.2 a ſingle figure, conſequently it would have
—— 10 for its *Denominator*, whereas by Vul-
.8 gar Fractions it ought to be 100; therefore there ought to be a cypher prefixt to it thus .08 by which means it is removed to the 2d place of Decimals, and conſequently the *Denominator* will be 100 as it ought.

S. Will this Rule for cutting off as many Decimal places in the product as you have Decimal places in both the Multiplicand and Multiplier hold good, when a Decimal is to be multiplied by integers: or when integers and Decimal parts are to be Multiplied by integers and Decimal parts?

Maſt. Yes, as I ſhall prove by the following caſes.

Caſe 1ſt. Parts multiply'd by parts, the product will be parts; as has already been prov'd both by *Vulgar Fractions* and *Decimals.*

Caſe 2d. Parts multiply'd by Integers, the product will ſometimes be parts, ſometimes Integers, and ſometimes Integers and parts.

1ſt. It will be parts, when the product of the figure in the firſt place of Decimals, multiply'd by the Integers, are under 10, as in this caſe. Multiply 213
by 4
——
.852

HERE you ſee the number of Decimal places are not increaſed in number, tho' increaſed in value; for .213 would have had 1000 for its *Denominator*, and

852 will have the ſame; wherefore $\frac{852}{1000}$ muſt be more invalue than $\frac{213}{1000}$.

2d. Parts multiply'd by Integers, the product will be Integers, when the product of the ſaid figure in the firſt place of Decimals multiply'd by the Integer amount to ten, or any equal number of tens, as for inſtance this. Multiply .500 by 8

$$\begin{array}{r} \text{Multiply } .500 \\ \text{by } 8 \\ \hline 4.000 \end{array}$$

HERE you ſee the 5 which ſtood in the firſt place of Decimals, having 10 for its *Denominator*, conſequently ſo many tens as the product of the two figures is, ſo many Integers muſt there be, ſo the product here being 40, it is therefore $\frac{40}{10} = 4$ Integers.

3d. Parts multiply'd by Integers the product will be Integers and parts, when the product of any of the Decimal places multiply'd by the Integer are not exactly compoſed of any equal number of tens, and the product of the figure in the firſt place of Decimals multiply'd by the ſaid Integers exceed 10, as in this inſtance.

$$\begin{array}{r} \text{Multiply } .742 \\ \text{by } 6 \\ \hline 4,452 \end{array}$$

Caſe 3d. Integers and parts multiply'd by Integers and parts, the product will be Integers and parts, which caſe needs no proving after what has been already ſaid.

Note, *In all thoſe ſeveral caſes it is plain the number of Decimal places in the product cannot be increaſed beyond thoſe in the Multiplicand and Multiplier. for when ever the product of the figure in the firſt place of Decimals, multiply'd by any of the multiplying Integers exceeds* 10 *then it becomes an Integer or Integers.*

Scho. Might not parts multiply'd by parts, give a product conſiſting of more figures, than both Multiplicand and Multiplier together, and conſequently give ſome Integers ?

Maſt. No, it's impoſſible; for let us ſuppoſe (for inſtance) the Multiplicand to be .99 and the Multiplier .9, which are the greateſt numbers which could be expreſs'd by the ſame number of figures, here you ſee the product cannot exceed three places, as will appear by the work. Multiply .99
by .9

.891 conſequently there being alſo three Decimal places in the Multiplicand and Multiplier, the product muſt therefore be all Decimal parts.

Scho. There is ſtill one thing more I would know; and that is in reducing $\frac{2}{5}$ and $\frac{5}{8}$ to a Decimal, why you particularly made choice of 10 and 1000 for the Denominators ?

Maſt. We cannot aſſign any particular *Denominator*, only we know it muſt be 10, 100, 1000, &c. therefore we add a cypher to the Dividend as we have occaſion till we find nothing remain.

Scho. Muſt we then carry on the Diviſion by adding cyphers till nothing remains ?

Maſt. Not always, for there are many *Vulgar Fractions* which cannot be reduced to a perfect Decimal; in ſuch caſes therefore, five or ſix figures in the quotient will come near enough the truth; as for inſtance, I want to reduce $\frac{2}{7}$ to a Decimal whoſe *Denominator* ſhall be 10, 100, 1000 &c. I therefore ſay,

$$\begin{array}{l} d \quad n \quad\ \ d \\ \text{If } 7 : 2 :: 100000 \\ \qquad\qquad\quad 2 \\ \hline \qquad\quad)\ 200000\ (\\ \hline \qquad\qquad 28571\tfrac{3}{7} \end{array}$$

here you ſee I have .28571 for a decimal, whoſe *Denominator* is 100000, which is not exactly equal to $\frac{2}{7}$ yet it comes near enough the truth; for the quotient conſiſting of five figures, the laſt figure towards the right hand is one hundred thouſand part of an integer, wherefore we looſe but $\frac{3}{7}$ of $\frac{1}{100000}$ part of an Integer: but when there happens to be no remainder, then is the quotient a decimal exactly equivalent to the Vulgar Fraction given.

S. But does not adding cyphers at pleaſure to your dividend alter the value of your quotient, ſince it will be greater or leſs, according as you increaſe your dividend?

M. No, for your *Denominator* will always be increaſed in theſame proportion, for as I obſerved before, if your decimal in the quotient conſiſts only of one figure it will have 10 for its *Denominator*, if of 2 figures 100, and if of three figures 1000 &c. Therefore cyphers annext to the right hand of decimals, neither add to, nor take from the value of it: but a cypher prefixt to the left of a decimal or decimals leſſens the value of it by moving it ſo many places towards the right hand, ſo .5 being the 1ſt place of decimals is $\frac{5}{10}$, but place a cypher before it thus 05 it becomes $\frac{5}{100}$, and again if we place two cyphers before it thus .005 it is $\frac{5}{1000}$.

⁂

CHAP. IV.

DIVISION of DECIMALS.

DIVISION of Decimals is alſo performed the ſame as in whole numbers, only you muſt remember when the work is finiſhed to cut off as many decimal places from the right hand of your quotient, as the Decimal places in your dividend exceed thoſe in your diviſor: if there be not ſo many figures in your quotient, then you muſt prefix cyphers to the left hand of it, to make it up that number.

2d When the decimal places in the dividend are exactly equal to thoſe in the diviſor, then your quotient will be all integers.

3d When there are not ſo many decimal places in your dividend as there are in your diviſor; then you muſt add as many cyphers into your dividend, as will make the decimal place in it equal to thoſe in the diviſor, which cyphers you muſt always eſteem as decimals, placing a point before them, to diſtinguiſh them from the integers, except when there is a decimal before them, and then the point would be needleſs.

Note. *If when you have added ſo many cyphers to your dividend as to make the decimal places in it, equal to thoſe in the diviſor, there ſhould happen to be no remainder, then is your quotient (as I obſerved before) all Integers, and the anſwer perfect: but if there ſhould be a remainder, eſpecially if a large one, then you muſt continue ad-*

ding cyphers to your dividend, till you have got 4 or 5 decimal places in your Quotient For ſuppoſing (when the decimal places in your dividend were exactly equal to thoſe of the diviſor) the remainder to have been 25 and the diviſor to have been 26, then you would have $\frac{25}{26}$ of 1 Integer. But if after adding more cyphers to your dividend, and proſecuting your work till you had got five decimal places in your quotient, there ſhould yet remain 25, then would it only be $\frac{25}{26}$ of 100000 part of an integer.

Note 2d. *By this means of adding cyphers you ſee we can (as in vulgar Fractions) divide a leſſer number by a greater, or even a Decimal by a whole number, as will appear by ſome of the examples following.*

Column 1ſt.] **EXAMPLES.** [Column 2d.

1ſt Divide .450 by .25

.25).450(1.8
25

200
200

...

Here the Decimal places in my Dividend being more than thoſe in my Diviſor by 1, I therefore cut off my 8 in the quotient.

2d. Divide 2.5000 by 40

40)2.5000(.0625
2 40

.100
80

.200
200

Here

3d. Divide 4.50 by .25

.25)4.50(18
25

200
200

...

In this 3d queſtion the Decimal places in the Dividend being equal to thoſe in the Diviſor, the quotient (according to the rule) are all Integers.

4th. Divide 9 by .9

.9)9.0(10
9

.0

In this 4th queſtion the Dividend being 9 Integers and the Diviſor 9 Decimal parts; I therefore add

a

Column 1ſt.]

Here when my Diviſor was finiſhed I had only three places in my quotient, and having 4 Decimal places in my Dividend and none at all in my Diviſor, I muſt therefore according to the general rule, have four Decimal places in my quotient, wherefore I am obliged to prefix a cypher before the other three to make that number you ſee.

[Column 2d.

a cypher to the Dividend and cut it off from the Integers by a point, that I may have as many Decimal places in my Dividend, as in my Diviſor; and then proſecuting my work, I have 10 Integers for the quotient; for it is plain that $\frac{9}{10}$ *muſt be contain'd in 9 Integers 10 times.*

5th. Divide 19.4 by 37.5
6th. Divide 47121 1 by 47
7th. Divide 41 8 by .1812
8th Divide .76121 by 41
9th. Divide 612812 by 7 21
10th. Divide .21819
by 721.

11th. Divide 9 by 7121
12th. Divide 9. by 9.
13th. Divide 14 by 47.2
14th. Divide 1. by 863.
15th. Divide .012181 by .2
16th Divide .0001212
by 018.

Scho. I ſhould be glad to know the reaſon on which the general rule is founded, for finding the Decimal places in the quotient, namely why you cut off as many Decimal places from the right hand of your quotient, as the Decimal places in your dividend exceed thoſe in your diviſor.

Maſt. Becauſe the quotient and diviſor in Decimals (as in whole numbers) being multiply'd one by the other, muſt produce the dividend: as therefore it has already been proved in Multiplication of Decimals, that the product muſt always have as many Decimal places, as both the Multiplicand and Multiplier have together: conſequenntly the dividend in Diviſion being equal to the product of the diviſor multiply'd by the quotient; the Decimal places of the quotient, and thoſe of the Diviſor together, muſt be equal to the Decimal places in the dividend.

CHAP.

CHAP. V.

REDUCTION of DECIMALS.

SECTION 1ſt.

When it is required to reduce a Simple Fraction to a Decimal of equal value.

Rule. Annex cyphers to the Numerator, and divide by the Denominator.

EXAMPLES.

1ſt. Reduce $\frac{3}{4}$ to a Decimal.

4) 3.20 (.75 = $\frac{3}{4}$
28
—
20
20
..

2d. Reduce $\frac{1}{2}$ to a Decimal.
Anſwer .5

3d. Reduce $\frac{1}{4}$ to a Decimal.
Anſwer .25

4th. Reduce $\frac{5}{8}$ to a Decimal.
Anſwer .625

5th. Reduce $\frac{1}{6}$ to a Decimal.

6) 1.0000 (.1666 +
6...
—
40
36
—
40
36
—
40
36
—
4

N.B. A croſs after the quotient, thus, (+) denotes the Decimal imperfect: for if I had annext cyphers for ever here, I ſhould ſtill have had 4 for a Remainder.

6th. Reduce $\frac{7}{8}$ to a Decimal. —— Anſwer .875

SECTION 2d.

When a Compound Fraction is given to be reduced to a Decimal.

Rule. Reduce it firſt to a Simple Fraction, and then proceed as in Section 1ſt.

S. I ſhould be glad to know the reaſon on which this Rule is founded; namely, Why we annex cyphers to the Numerator, and divide by the Denominator?

M. This is only an abridgement of the general rule given, to find a Numerator inſtead of a Numerator of any given Fraction, which ſhall bear ſuch proportion to 10, 100, 1000. &c. as the ſaid given Numerator had to its Denominator. For inſtance, were it required to reduce $\frac{3}{4}$ to a Fraction of the ſame value, whoſe Denominator ſhould be 100; then according to the general rule it would be found, thus, Denom. Num Denom.
If 4 : 3 : : 100
here according to the Rule of Three, we would multiply 100 by 3. and divide by 4. Now it is plain, that as the Denominator of a Decimal Fraction muſt always be 10, 100 1000, &c. that multiplying an unit with any number of cyphers (whatever) annext to it, by any other number is the very ſame thing, as annexing cyphers to the ſaid number; for multiplying 1000 by three, is equal to 3 cyphers annext to 3.

Note, In reducing a Vulgar Fraction to a Decimal, both Numerator and Denominator are to be eſteemed Whole Numbers.

SECTION 3d.

When it is required to reduce any known part of an Integer, to a Decimal of the ſame Integer; or to the Decimal of an Integer of a higher Denomination.

GENERAL RULE.

If a ſimple Number, make it firſt a Vulgar Fraction of that Integer you would have the Decimal of, and then proceed as in Section 1ſt. For inſtance, if you would reduce *6d* to the Decimal of a ſhilling: then the Vulgar Fraction would be $\frac{1}{2}$: but if you would have it the Decimal of a pound, then it would be $\frac{1}{40}$. 2

2d. If the given parts conſiſt of divers Denominations, which is an alliquot part of the Integer you would have the Decimal of, ſuch as 6*s* 8*d*, and the Integer, a pound, then it will be $\frac{1}{3}$ of a pound; but if it is no alliquot part, ſuch as 12*s* 6*d* $\frac{3}{4}$, then reduce it into the loweſt name given, viz. Farthings, for a Numerator, and place the farthings in a pound, viz. 960, for a Denominator, and then proceed as in Section 1ſt.

Note 1ſt. *The decimal of* $\frac{1}{4}$ *of any thing, will always be* .25 *the decimal of* $\frac{1}{2}$ *will be* .5 *and the decimal of* $\frac{3}{4}$ *will be* .75.

S I cannot yet conceive how the operations can be ſhortened by the uſe of decimals, except where you can immediately know the decimal which repreſents the ſum, or fraction at ſight, ſuch as are expreſſed in the laſt note, viz. $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$; for in many caſes (where the parts to be reduced to a decimal conſiſt of divers denominations) ſuch as in the 2d rule in the 3d ſection, viz 12*s*. 6*d*. $\frac{3}{4}$ the trouble of finding the decimal and the number of figures are almoſt equal to the operation by whole numbers.

M. Suppoſe even that to be granted, yet in all queſtions in plural proportion (I mean when there are more ſtatings than one) the operation by decimals are not only much ſhorter, but more exact as I ſhall prove in a proper place; but beſides that there are decimal tables, which moſt merchants make uſe of, which ſhew the decimal of any of the known parts of money, weight, meaſure &c. of any Integer of the ſame kind, although they may be almoſt as readily found by the following method. By which let us again reduce 12*s*. 6*d*. $\frac{3}{4}$ to the decimal of a pound.

FIRST then find the decimal of the loweſt denomination given, equal to an integer of the next ſuperior denomination, which in this example is $\frac{3}{4}$) therefore according to the 1ſt note in this ſection ¾d. is .75 decimal of a penny, I therefore place .75 after my 6d. in this manner 6.75 which is 6d. and 75 decimals of a penny.

I next divide that by 12, adding cyphers as I find occaſion thus,

d
12) 6.7500 which gives the decimal
of a ſhilling = 6d. ¾ viz.
.5625 .5625.

I then place my 12s. to the left hand of my point thus, 12.5625 which is 12s and 5625 decimals of a ſhilling and dividing it by 20, adding cyphers occaſionally as before, I have the decimal of a pound = 12s. 6d. ¾. See the whole operation together

12) 6.7500 (
2|0) 12.56250 (0

.6 28125 of a pound.

Note 2d. *In dividing by 20 here, I add a cypher more than I had occaſion to make uſe of, for the cyphers cut off in my diviſor, and therefore according to the rule of dividing by whole numbers I ſhould have ſet a nought alſo in my dividend, but as cyphers placed to the right hand of a decimal, do not in the leaſt alter the value of them, I therefore omit it. You may likewiſe find the decimal of any of the known parts of weights, meaſure, &c. by the ſame method.*

S. This is indeed a much ſhorter way of finding the decimal equal to any given quantity; but I cannot ſee the reaſon of the operation.

M. That I ſhall ſhew you in the next ſection; but firſt I ſhall give you ſome more examples in this.

2d Reduce 7s 6d. to the decimal of a pound.
Anſwer .375l.

3d Reduce 10*s*. 9*d*. ¼ to the decimal of a pound.
Anſwer .5385416+

4th Reduce 3 farthings to the decimal of a ſhilling.
Anſwer .625 *ſhill.*

5th Reduce 24 grains to the decimal of a *lb.* troy.
Anſwer .0041666 + *lb.*

6th Reduce 12 drams to the decimal of a *lb.* avoirdupoiſe. *Anſwer* .046875 *lb.*

7th Reduce 4 *cwt.* 2*qrs.* to the Decimal of a tun.
Anſwer .225 *tun.*

8th Reduce 3 *qrs.* 2 *nails* to the decimal of a yard.
Anſwer .875 *yard.*

9th Reduce 4 *perches* to the decimal of an acre.
Anſwer .025 *acre.*

10 Reduce 10 gallons to the decimal of a hogſhead.
Anſwer .15873 *hhd.*

11th Reduce 7 minutes to the decimal of a day.
Anſwer .0048611 + *day.*

12 Reduce 72 days to the decimal of a *year.*
Anſwer .1972602 + *year.*

SECTION 4th.

TO find the value of a decimal Fraction in the known parts of an integer.

Rule. Multiply the decimal by the parts of the Integer, and cut off ſo many places towards the right hand as your decimal conſiſts of, and thoſe to the left, will be integers of the next inferior denomination.

EXAMPLES.

1ſt What is the value of .628175 of a pound ſterling?
Anſwer 12*s*. 6*d*. ¼.

See the work .628175
20

s 12.563500
12

d 6.762000
4

far. 3.048000

2d What is the value of .484374 of a pound?
Anſwer 9*s.* 8*d.* $\frac{1}{4}$.
3d What is the value of .585 of a ſhilling?
Anſwer 7*d.*
4th What is the value of .3134 of a lb. troy?
Anſwer 6 *oz.* 3 *drs.* 5 *grs.*
5th What is the value of .875 of a cwt.
Anſwer 3 *qrs.* 14 *lb.*
6th What is the value of .45 of a yard.
Anſwer 1 *qrs.* 3 *nails.*
7th What is the value of .365 of an acre?
Anſwer 1 *rood,* 18 *perches,* 2 *yds.*
8th What is the value of .6354 of a foot.
Anſwer 7 *inches,* 1 *b. c.*
9th What is the value of .48375 of a year?
Anſwer 6 *months,* 8 *days,* 2 *hours.*

S. This operation is very ſimple, but the reaſon on which it is founded, I don't underſtood.

M. The reaſon is as plain, as the operation is ſimple; and is the ſame thing with that already given for reducing a ſimple fraction to its proper value, as I ſhall make appear by the 1ſt example in this ſection, where the value of 628175 of a pound is required. Now according to the definition already given of a decimal fraction, the *Denominator* is always underſtood to be an unit, with ſo many cyphers annext to it, as there are places in the *Numerator*; wherefore the *Denominator* of 628175 muſt be 1000000; ſo that the ſaid decimal being converted into a vulgar Fraction would ſtand thus $\frac{628175}{1000000}l$ the rule for finding the value of which is this, Multiply the *Numerator* by 20, and divide by the *Denominator*; and again multiply the remainder (if any) by 12, and divide the product again by the *Denominator*, and ſo on.

Now as dividing by 10, 100, 1000, &c. &c. &c. is no
more

more than cutting off as many figures to the right hand of your dividend as there are cyphers in your diviſor; and the cyphers in the Denominator of a Decimal Fraction (which is eſteem'd your Diviſor) being always equal to the number of places in your Numerator or Multiplicand; therefore after you have multiplied your Numerator or Decimal by the common parts of the Integer, you cut off as many Decimal places to the right hand of the product, as there were places in the given Decimal, which is the very ſame thing as if you had divided by the Denominator.

THE reaſon of this rule being fully prov'd; the reaſon for the rule given in ſection 3d will appear evident. For if multiplying .628175 Decimals of a pound by 20 gives 12 ſhillings, and 563500 Decimal parts of a ſhilling; conſequently dividing 12.563500 ſhillings by 20 muſt give the Decimal of a pound; Multiplication and Diviſion being exactly the reverſe of each other.

BUT the value of any Decimal parts of money may more readily be found (by inſpection only) thus,

THE figure which ſtands in the firſt place of Decimals (being ſo many tenth parts) will be juſt double the number of ſhillings; the figure in the 2d place of Decimals (if 5) will be one ſhilling more, $\frac{5}{100}$ being equal to one ſhilling If the 2d figure be above 5, then ſuppoſe the overplus to be placed before the 3d figure and it will be ſo many farthings; if the 2d figure be under 5, you muſt carry it to the 3d figure in the ſame manner, and eſteem it as ſo many farthings, as before.

Note 1ſt. *For every 24 farthings; you muſt abate 1, which will come near enough the truth.*

So this Decimal .628175 being valued according to this method, I firſt double my firſt figure 6 which is 12s. then becauſe my figure in the 2d place of Decimals is only 2; I therefore take it, and my 8 together, which

I call 28 farthings, from which taking 1 (becauſe it is above 24) I have 27 farthings = 6$\frac{3}{4}$*d.* ſo that I find the whole value of the Decimal to be 12*s.* 6$\frac{3}{4}$*d.* as before.

Again in valuing this Decimal .79921, firſt 7 being doubled is 14*s.* then becauſe my ſecond figure exceeds 5, I therefore add 1 ſhilling more which makes 15*s*; and the 4 (which my 2d figure is above 5) I ſuppoſe to be placed before my 3d place of Decimals, which makes 49, in which 24 being contain'd twice, I therefore abate 2 farthings; ſo that I have 47 farthings = 11$\frac{3}{4}$*d.* which together with the 15 ſhillings makes the whole value to be 15*s.* 11$\frac{3}{4}$*d.*

Scho. Pray what is the reaſon you abate 1 farthing for every 24?

Maſt. Becauſe the 3d place of Decimals (as I have often obſerved to you before) has always 1000 for its Denominator; now if we ſuppoſe the Numerator to be 24, its plain that $\frac{24}{1000}$ cannot be equal to 24 farthings; but had the Denominator inſtead of 1000, been 960: then would it been exactly 24 farthings. And therefore as $\frac{25}{1000}$ is = $\frac{24}{960}$ we abate a farthing for every 24.

CHAP. V.

INTEREST, by DECIMALS.

AS the operation by decimals is the ſame with that in whole numbers, not only in the Single and Double Rules of Three; but in all other rules depending on them (ſuch as Fellowſhip, Barter, Diſcount, &c. &c.) regard being had to Multiplication and

Diviſion of Decimals, as alſo to the manner of reducing them, I think it needleſs to repeat thoſe rules again here, as the learner can be at no loſs when or how to apply them. I ſhall therefore only ſhew him their particular uſe in finding the Simple Intereſt of any ſum for any number of days, and in Compound Intereſt, Rebate, &c. give him no more examples than what are abſolutely neceſſary, to make him underſtand the calculation of annuities, and the conſtruction of tables for that purpoſe. And firſt of

SIMPLE INTEREST for *Days.*

TO find the ſimple Intereſt of any ſum of money for any number of days.

Rule. Multiply the intereſt of 1 pound for one day, at the given rate, by the principal, and by the number of days; the laſt product is the intereſt required.

THE following table ſhews the intereſt of 1*l.* for a day, at any rate from 1 to 6 per cent.

The TABLE.

per cent.
1 is = .00002739726
2 is = .00005479452
3 is = .00008219178
4 is = .00010958904
5 is = .0001369853
6 is = .001643836

EXAMPLES.

1ſt What is the Intereſt of 120*l.* for 126 days, at 4 per cent? *Anſwer* 1*l.* 13*s.* 1*d.* ½ +.

2d What is the intereſt of 126*l* for 145 days, at 6 per cent? *Anſwer* 3*l.* 0*s.* 0*d.* ¾

3d What is the intereſt of 200*l.* from the 1ſt of June 1770, to the 9th of March following, at 5 per cent? *Anſwer* 7*l.* 13*s.* 11*d.* ½

Note 1ſt *When the rate is 5 per cent, there is a better method of finding the intereſt of any ſum for any number of days, which is by this*

Rule. Multiply the Principal by the number of days, and divide the product by 7300, or (cutting off the two cyphers) by 73, and the quotient will be the anſwer.

4th What is the intereſt of 355*l.* for 25 days, at 5 per cent per annum?

Anſwer 1*l.* 4*s.* 3*d.* ¾

See the work.

```
              £
             355
              25
            ----
            1775
            710
           ------
   73|00) 8875000|00 ( 1.21575
          73·····          20
          ---          -------
          157          4.31500
          146               12
          ---          -------
           115         3.78000
            73               4
           ---         -------
           420         3.12000
           365
           ---
            550
            511
            ---
             390
             365
             ---
              25
```

Note 2d. *The decimal in the quotient might have been valued by inſpection.*

IN performing this queſtion, I firſt multiply my 355l. by 25 days. and although the product be large enough to contain the Diviſor (7300) once, yet as there happens a large remainder, I therefore add cyphers to my dividend in order to carry my quotien's out to 4 or 5 figures, which makes the remainder at laſt of no ſignification.

5th A borrowed of B 500*l.* of which he paid him in part at the end of 25 days, 150*l.* 10*s.* and 40 days after that, he paid him 80*l.* and 20 days after that he paid him 170*l.* more. I demand what remained due to B. *Anſwer* 103*l.* 17*s.* 3*d.* ¾

```
1ſt500     2d 500            3d349.5          4th 349 5
   25         150   10            40                80
 ----       ---------        -------              -----
 2500         349   10       13980.0              269.5
1000                                                 20
-----             5th 12500                       -----
12500                 13980 0                    5390.0
                       5390.0
           73|00) 31870 00|00 (4.3657
                  292             -------
                  ----           4l. 7s. 3d ¾
                   267              £
                   219          6th 500   0   0
                  -----               4   7   3 ¾
                    480           ---------------
   £     s          438           504   7   3 ¾
7th 150  10        -----          400  10
     80             420          -----------------
    270             365        rms. 103  17   3 ¾
   -----            ---
  £400   10         550
                    511
                   -----
                     39
```

TO ſolve this queſtion, I firſt multiply 500*l.* by 25 [illegible] days (which was the time it continued in A's hand bebefore any payment was made) 2d. I ſubtract 150*l*. 10*s*. (the 1ſt payment) from 500*l*. and the remainder is 349*l*, 10*s*. which continued 40 days; therefore, 3dly, I multiply 349.5*l*. by 40 days; 4th. I ſubtract 80*l*. (the next payment) from 349.5*l*. and the remainder 269.5*l*. I multiply by 20 days, which was the time it continued before the 170*l*. was paid. 5th I add the ſeveral products together, and divide the whole ſum by 7300, and the quotient 4*l* 7*s*. 3*d* ¼ is the intereſt of the ſeveral ſums for the reſpective times which they continued in A's hand: 6th. I add the intereſt ſo found to the 500*l*. 7th. I add the ſeveral payments together, and laſtly ſubtracting that ſum from the principal and intereſt, the remainder ſhews what A then ſtands indebted to B, viz. 103*l*. 17*s*. 3*d* ¼.

THIS laſt operation will ſufficiently prove to the learner, how much Decimals are preferable to whole numbers in queſtions of this kind: for by working this example by whole numbers, you muſt not only have had a ſtating in order to find out the intereſt of every particular ſum, according to its reſpective time: but the remainder in each ſtating would be laſt, and conſequently give an imperfect anſwer, whereas by this method the ſums of the ſeveral products, being reſerved till the laſt, and divided all at once, there can only be one remainder, which may be rendered inſignificant by adding cyphers to your Dividend, and carrying the Decimal places inyour quotient to the length of 4 or 5 figures, as I have proved before.

Scho. This is indeed a very commodious way of finding the intereſt of any ſum, for any number of days; but I ſhould be glad to know the reaſon of the operation?

Maſt. It is no more than a contraction of the rule of five, as I ſhall prove by the following example.

What is the intereſt of 500*l.* for 25 days at 5 per cent per annum? Which being ſtated according to that rule would ſtand
£ *dys. int.* £ *dys.*
Thus, If 100 : 365 : 5 :: 500 : 25

Read thus, If 100*l* in 365 days (which is the days in a year) gain 5*l.* intereſt, what intereſt will 500*l.* gain in 25 days.

HERE both ſtatings being direct, according to the rule for working in that caſe, the three laſt terms are to be multiply'd into one another for a dividend, and the two firſt together for a diviſor.

LET us then ſuppoſe the two laſt only to be multiply'd together, and alſo the two firſt together, now it is plain, that whether I multiply the product of the two laſt, again by the middle term 5 (according to the rule) and divide that product, by the product of the two firſt; or whether I divide the product of the two firſt by the ſaid middle term 5, and by that quotient divide the product of the two laſt, it muſt give the ſame anſwer; for the product of the two laſt, being made 5 times more, muſt bear the ſame proportion to the product of the two firſt; as the product of the two firſt, made 5 times leſs, will bear to the product of the two laſt.

As therefore the two firſt terms, muſt always be the ſame in all queſtions where the intereſt of any ſum for any number of days is ſought, namely 100*l.* and 365 days, ſo conſequently the quotient ariſing from the product of the two firſt, divided by 5, viz. 7300 muſt be a ſettled Diviſor at 5 per cent. for 365 multiply'd by 100, and the product divided by 5, gives the quotient 7300 as before mention'd.

Scho. But what muſt be done when the product of the two laſt will not contain the diviſor 7300.

Maſt. Add cyphers to your dividend, which muſt be accounted ſo many Decimal places, as directed before.

CHAP. VI.

COMPOUND INTEREST, *by* DECIMALS.

NOtwithſtanding *Compound Intereſt* may be performed with far leſs trouble, in much fewer figures, and alſo more exactly, by Decimals, than by Whole Numbers; yet at beſt the operations are tedious: For which reaſon, moſt Merchants make uſe of Tables for all calculations of that kind. I ſhall therefore inſert ſuch as are of the moſt general uſe, and alſo ſhew you the Principles on which they are conſtructed. And altho' they will not be calculated for ſo many *Years*, or at ſo many various Rates per *Cent.* as you may have occaſion for, yet they will be ſufficient to ſhew the uſe of them.

TABLE I.

Shewing what 1 pound *will amount to, being forborne to any term of* Years *under* 16, Compound Intereſt *being computed yearly, at any of theſe* Rates, *viz.* 4, 5, 6, 7, *and* 8, *per* Cent. *per* Annum.

Years	4	5	6	7	8
1	1.04000	1.05000	1.06000	1.07000	1.08000
2	1.08160	1.10250	1.12360	1.14490	1.16640
3	1.12486	1.15762	1.19101	1.22504	1.25971
4	1.16985	1.21550	1.26247	1.31079	1.36048
5	1.21665	1.27628	1.33823	1.40255	1.46932
6	1.26531	1.34009	1.41851	1.50073	1.58087
7	1.31593	1.40710	1.50363	1.60578	1.71382
8	1.36856	1.47745	1.59384	1.71818	1.85093
9	1.42331	1.55132	1.68947	1.83845	1.99900
10	1.48024	1.62880	1.79084	1.96715	2.15892
11	1.53945	1.71033	1.89829	2.10485	2.33163
12	1.60103	1.79585	2.01219	2.25219	2.51817
13	1.66507	1.88564	2.13292	2.40984	2.71962
14	1.73167	1.97993	2.26090	2.57853	3.93719
15	1.80094	2.07892	2.39655	2.75903	3.17216

The Conſtruction of the preceeding Table.

The numbers 1, 2, 3, 4, &c to 15, in the firſt column on the left hand ſignify years; the numbers 4, 5, 6, 7, 8, placed at the head of the reſt of the columns ſignify rates of intereſt for 100*l*, lent for a year, and the numbers placed in the ſeveral columns underneath thoſe rates of intereſt, are found out by the Rule of Three in Decimals in manner following, viz.

ſtating 1ſt. If 100 : 104 : : 1 *Anſwer* 1.04
2d. If 100 : 104 : : 1.04 d° 1.0816
3d. If 100 : 104 :: 1.0816 d° 1.12486

THAT is to ſay, Firſt if 100*l*. in 1 year, at 4*l*. per cent, amount to 104*l*. what will 1*l*. amount to in the ſame time, and at the ſame rate? *Anſwer* 1.04*l*. which 1.04 or 1.04000 (the cyphers after the 4 being of no value in Decimals) is the 1ſt number in the column under 4 per cent. and is placed right againſt 1 year in the firſt column.

Secondly, I ſay, If 100*l*. in 1 year amount to 104*l*. what will 1.04*l*. (which is the principal and intereſt of 100*l*. for the 1ſt year) amount to at the 2d year's end at the ſame rate?— *Anſwer* 1.0816*l*. which anſwer is the 2d number in the ſaid column of 4 per cent, and is placed right againſt 2 years in the 1ſt column.

Thirdly, I ſay, If 100*l*. in 1 year amount to 104*l*. what will 1.0816*l*. amount to: and find the anſwer to be 1.124864 which is the 3d number in the column of 4 per cent, and is placed right againſt 3 years in the 1ſt column. Hence it appears that 1*l*. at 4*l* per cent per annum compound intereſt, will at the end of 3 years be increaſed to 1.124864*l*. = 1*l* 2*s* 5*d*. $\frac{1}{4}$. After the ſame manner the reſt of the numbers in the other columns are found out.

The uſe of TABLE 1ſt.

Queſt. What will 325*l* 15*s*. amount to in 7 years, at 5 per cent per annum compound Intereſt?

Anſwer 458*l*. 7*s*. 3*d* and is thus found:

Firſt looking into that column in the ſaid 1ſt table, under 5 per cent & right againſt 7 years; I find the number 1.40710, which ſhews that 1*l.* being forborne 7 years, at 5 per cent per annum, compound intereſt, will then amount to 1.40710*l.* Therefore after having reduced my 15 ſhillings in the queſtion to a decimal, which is .75, I multiply the ſaid tabular number viz. 1.40710*l.* by 325.75*l.* (the ſum given in the queſtion,) and the product is 458.3628250*l.* = 458*l.* 7*s.* 3*d.*

Note. *If the Compound Intereſt of the ſaid* 325*l.* 15*s.* *had only been required, then ſubtract the ſaid ſum from the amount and the difference would have been the anſwer.*

TABLE II.

Shewing what 1 pound *Annuity, payable by yearly payments, will amount to. being forborne to the end of any term of years under* 16, Compound Intereſt *being computed at any of theſe rates,* viz. 4, 5, 6, *and* 7, *per* Cent. *per* Annum.

Years	4.	5.	6,	7.
1	1 00000	1 00000	1.00000	1.00000
2	2 04000	2 50000	2 06000	2.07000
3	3·12160	3 15250	3 18360	3.21490
4	3·24646	4 31012	4 37461	4 43094
5	5·41632	5 52563	5 63709	5 75[illegible]
6	6.63297	6 80190	6 97531	7 15329
7	7 89829	8 1,200	8.39383	8 65402
8	9 21422	9.64900	9 89746	10 25980
9	10.58279	11.0265[illegible]	11 4913[illegible]	11 97798
10	12 00610	12 55789	13·18079	3 81644
11	13 48635	14 20678	14 97164	15·78359
12	15 02580	15.91712	16 86994	17 88845
13	16.62683	17.71298	18 88213	20 14064
14	18·29191	19.59863	21.01506	22 55048
15	20 02358	21 57856	22.27596	25·12902

Queſt. Suppoſe an annuity of 300*l*. payable at yearly payments, be forborne, and all unpaid until the end of 4 years, the queſtion is, what then will be due, Compound Intereſt being computed at the rate of 6 per cent per annum for each yearly payment, from the time it becomes due, to the end of the ſaid term of 4 years.

FROM the nature of the queſtion it is evident, that there muſt be computed what 300*l* due at the firſt years end will amount to in the 3 following years, at 6 per cent compound intereſt. Alſo what 300*l* due at the 2d years end will amount to in two years, viz. the 3d and 4th years; likewiſe what the 300*l*. due at the 3 years end will amount to in one year, viz. the 4th year: all which ſums being added to the 300*l*. due at the 4th year's end, (which is incapable of any further improvement) the ſum will be the whole money due at the end of the 4th year, viz. 1312.3848*l*. = 1312*l*. 7*s*. 8*d*. $\frac{1}{4}$ as will appear from the following operation.

Firſt 300 due at the 1ſt years end, will in 3 years be increaſed to	357.3048
Secondly, 300*l*. due at the 2d years end, will in two years amount to	337.08
Thirdly, 300*l*. due at the 3d years end, will in 1 year be augmented to	318.
And fourthly, the laſt payment of the annuity due at the end of the 4th year is	300.

the whole ſum due at the end of the 4th year £ 312.3848

Upon theſe principles the 2d table aforegoing is conſtructed; only inſtead of 300*l* you ſeek the amount of 1*l*. annuity in the ſame manner, for any term of years, and at any rate per cent mentioned in the ſaid table.

The Uſe of TABLE 2d.

FIRST find the amount of 1*l* annuity at the rate per cent, and for the time given in the queſtion, according to the directions given as in table 1ſt, and then multiplying the ſaid amount of 1*l.* by the annuity whoſe amount is required, the product will be the anſwer.

TABLE III.

Shewing what 1 pound, *payable at the end of any term of Years to come, (under* 16) *is worth in* Ready Money, Diſcount *or* Rebate *being yearly computed at any of theſe Rates. viz.* 4, 5, 6, 7. & 8, *per* Cent *per* Annum, Compound Intereſt.

	4	5	6	7	8
1	.961538	.952381	.943396	.934579	.925925
2	.924556	.907029	.889996	.873438	857338
3	.888996	.863837	.839619	.816297	.793832
4	.854802	822702	.792095	.762895	.735029
5	.821927	783526	747258	712986	.680583
6	.790314	.740[illegible]5	.400960	.606342	.630169
7	.759917	710681	.665057	.622749	.583490
8	730690	.676839	627412	.552009	.540268
9	.702586	.644608	591898	.543933	.540248
10	.67556[illegible]	.613913	.558391	.508249	.[illegible]63103
11	.049580	.584679	520787	472092	.428882
12	.624596	.586837	496989	.444012	.397113
13	.600573	,530321	.468839	414964	.367697
14	.577474	.505067	442300	387817	.340461
15	55526[illegible]	481017	417265	362446	.315241

Queſt. 1ſt. Suppoſe 378*l.* 14*s.* 10*d.* were due 4 years hence, I demand what it is worth in ready money, rebate being made at 6 per cent per annum, Componnd Intereſt?

HERE it is evident by the queſtion that the whole money due at the 4 years end, will be leſſened in one year, viz. at the end of the 3d year, to 357 30,8*l.* (Rebate being made at 6 per cent) again that ſum will be leſſened in 1 year more, viz. at the end of the 2d year, to 337 08, which ſum at the end of the 1ſt year, will be again reduced to 318*l.* and laſtly rebating at 6 per cent as before for 1 year, the preſent worth will be found to be 300*l.* which will be manifeſt by the 4 following ſtating in the Rule of Three.

1ſt If 106 : 100 : : 378.743088
Anſwer 357 3048*l.*
2d If 106 : 100 : : 357.3048 *Anſwer* 337.08*l.*
3d If 106 : 100 : : 337.08 *Anſwer* 318*l.*
4th If 106 : 100 : : 318. *Anſwer* 300*l.*

SO that it appears from the foregoing operation, that 378*l.* 14*s.* 10 ¼ due four years hence, will be worth 300*l.* preſent money.

UPON this ground the preceeding table 3d is calculated, to ſhew what one pound due at the end of any number of years to come, is worth in ready money, rebate being made at any of the rates of compound Intereſt, mentioned in the ſaid table.

Conſtruction of the preceeding Table 3d.

£ £ £
1ſt If 104 : 100 : : 1
Anſwer .9615384615 &c.
2d If 104 : 100 : : .9615384615
Anſwer .9245562 &c.
3d If 104 : 100 : : .9245562.
Anſwer .888996+.

THAT is to ſay, Firſt, if 104*l.* decreaſe to 100*l.* in one year; what will 1*l.* decreaſe to in the ſame time? *Anſwer* .9615384615+; ſo that 961531 (for there is no neceſſity for more than 6 decimal places) is the firſt decimal in the column under 4 per cent, and placed right againſt 1 year.

Secondly, I ſay in like manner, If 104*l.* decreaſe to 100*l.* what will .9615384615 decreaſe to; anſwer

.9245562 &c. the firſt 6 places of which are the 2d decimal in the ſaid column of 4 per cent, and which is placed right againſt 2 years.

Thirdly, If 104*l.* decreaſe to 100*l.* what will .9245562 decreaſe to; which I find to be .888996+ which is the 3d decimal in the column of 4 per cent. Hence it appears that 1*l.* due at the end of 3 years to come is worth .888996+ = 17*s.* 9*d.* ¼ and ſomewhat more, in ready money, rebate being made at the rate of 4 per cent per annum compound Intereſt?

The Uſe of TABLE 3d.

Suppoſe you wanted to know how much ready money would diſcharge a debt of 378*l.* 14*s.* 10¼ due at the end of four years to come, rebate being made at 6 per cent per annum compound intereſt.

FIRST (after having reduced your 14*s.* 10¼*d* to a Decimal) look in your table right againſt 4 years and under 6 per cent, and you will find this Decimal .792095 which is the ſum which 1 pound, due at the end of four years to come, would be worth in ready money, which multiplying by 378.743088*l.* you have the anſwer 300*l* as before.

Note, *This proves the example given, to ſhew the principles on which* Table 1ſt *is calculated.*

* * *

TABLE

TABLE IV.

Shewing the preſent Worth of 1 pound *Annuity, to continue any term of Years under* 16, *and payable by yearly Payments*, Compound Intereſt *being computed at any of theſe Rates, viz.* 4, 5, 6, 7, & 8, *per* Cent *per* Annum.

Yea	4	5	6	7	8
1	.96153	.95238	.94339	.93457	.92592
2	1.88609	1.85941	1.83339	1.80801	1.78326
3	2.77509	2.72324	2.67301	2.62431	2.57709
4	3.62989	3.54595	3.46510	3.38721	3.31212
5	4.45182	4.32947	4.21236	4.10010	3.99270
6	5.24213	5.07569	4.91732	4.76653	4.62287
7	6.00205	5.78637	5.58233	5.38928	5.20636
8	6.73274	6.46321	6.20979	5.97129	5.74663
9	7.43533	7.10782	6.80169	6.51523	6.24688
10	6.11089	7.72173	7.36008	7.02358	6.71008
11	8.76047	8.30641	7.88687	7.49867	7.13896
12	9.38507	8.86325	8.38384	7.94268	7.53607
13	9.98964	9.39357	8.85268	8.35765	7.90377
14	10.56312	9.89864	9.29498	8.74546	8.24423
15	11.11838	10.37965	9.71224	9.10791	8.55947

THE finding out the preſent worh of an annuity is grounded upon this foundation, viz, If the preſent money which is paid for the purchaſe of an annuity, to continue any term of years, be put forth at any rate of compound intereſt, and all forborn untill the end of the ſaid term: that the total ſum then due, ſhall be equal to the whole annual payments of the annuity, compound intereſt being computed upon each payment as it becomes due to the end of the ſaid term, and at the ſame rate per cent.

FOR inſtance let it be required to find the preſent worth of an annuity of 378.743088*l.* to continue 3 years, compound intereſt being computed at 6 per cent per annum. *Anſwer* 1012*l.* 7*s.* 8 ½*d.*

IT is evident by the queſtion that there muſt be computed, the preſent worth of 378.743088*l.* due at the 1ſt years end, alſo the preſent worth of the ſame ſum due at the 2d years end, and in like manner for the 3d year; all which particular preſent values being added together, the ſum will be the preſent worth of the annuity propoſed; found in this manner.

£ £
1ſt. If 106 : 100 :: 378.743088*l.*
Anſwer 357.3048*l*

2d. If 106 : 100 :: 357.3048
Anſwer 337 08

3d. If 106 : 100 :: 337.08
Anſwer 318.

Therefore the total preſent worth of an annuity of 378.743088*l.* to continue three years is, } £1012.3148.
=1012*l.* 7*s.* 8¼*d*

THE Conſtruction of Table 4th depends upon the ſame principles, only you make 1*l.* your laſt term in your firſt ſtating; the anſwer to the firſt ſtating, your laſt term in your 2d ſtating; and the anſwer to your 2d ſtating, the laſt term in your 3d ſtating; and the anſwers ſo found will be the 3 ſeveral Decimals in your column (under the rate per cent you compute your rebate at) and right againſt 1, 2, and 3 years, in your firſt column

Note, *The manner of ſolving queſtions by this table is the ſame with that already ſhewn, for uſing the preceeding ones.*

TABLE

TABLE V.

Shewing what Annuity, payable by yearly Payments, to continue any term of Years under 16, 1 pound will purchaſe; Compound Intereſt *being computed at any of theſe Rates, viz.* 4, 5 6, 7, & 8, *per* Cent *per* Annum.

	4	5	6	7	8
1	1.04000	1.05000	1.06000	1.07000	1.08000
2	.53019	.53780	54 43	.55309	56076
3	.36034	36720	.37411	.38105	.38803
4	.27549	.28209	28859	.29519	.30192
5	22462	.23097	.2373	.24389	250,5
6	.19076	.19701	.2033	.20979	.21631
7	.16660	.17281	.17913	.18555	.19207
8	.14852	.15472	16103	16746	.17401
9	.13449	.14069	.14702	.15348	.16 07
10	.12329	1295	.13586	.14237	14902
11	.11414	12038	.12070	.133 5	.14 07
12	.10655	11282	.11927	.125 0	13269
13	.10010	.10645	.10296	.11965	.12652
14	.09466	.10102	10758	.11434	.12129
15	.08994	09634	10296	.10979	.116 2

Upon the following Principles, the Conſtruction of the foregoing Table 5th is grounded.

FIRST you may preſuppoſe any annuity at pleaſure at a certain rate per cent, and for the number of years wanted (as for inſtance 1*l* for 2 years at 6 per cent per annum. Next find by (table 4th) what the ſaid one pound annuity to continue 2 years at 6 per cent is worth in ready money. which by the ſaid 4th Table will be found 1.83339*l*. Then ſay by the rule of Three,

£ £ £
If 1 83339 : 1 : : 1

That is to ſay, If 1 83339*l* buy an annuity of 1*l*. to continue 2 years, at 6 per cent; what annuity will 1*l*. buy for the ſaid time and at the ſaid rate of intereſt, which by the operation will be found .54543*l.* = 10*s.* 11*d.* very near. Therefore the ſaid decimal .54543*l.* is to be placed as the 2d number in table 5th, under 6 per cent right againſt 2 years. Hence it follows that if 1*l.* be divided by every one of the numbers, in all the columns in the ſeveral rates per cent in table 4th, the quotients will give the reſpective numbers in the like columns in table 5th. In which operation it will be requiſite, that the numbers in table 4th be continued to more places than are there expreſſed, to prevent errors that may ariſe by adding of defective decimals.

Note. *The manner of ſolving queſtions by this table, is the ſame as in all the reſt.*

CHAP. VI.

OF SUPERFICIAL MEASURES.

THIS may be performed three ſeveral ways, viz. by Decimals, by Alliquot parts, and by Croſs Multiplication.

THE laſt of theſe methods is uſed by moſt workmen, except glaziers, whoſe foot being decimally divided, caſt up the contents of their work by decimals. However as the operation by alliquot parts is not only much more ſimple but in many caſes more conciſe than by croſs Multiplication; I ſhall therefore ſhew how the ſuperficial contents of any piece of work is to be found by Practice, glaziers work only excepted, which I ſhall perform decimally for the reaſons above given.

The

The TABLE.

12	*Thirds marked thus'''*	*make*	1 *ſecond, marked thus''*		
12	*Seconds*	*make*	1 *part,*	*marked*	*pts.*
12	*Parts*	*make*	1 *inch,*	*marked*	*in.*
12	*Inches*	*make*	1 *foot,*	*marked*	*feet.*
9	*Square feet*	*make*	1 *ſquare yard.*		

GENERAL RULE.

Multiply the length by the breadth, and the product is the ſuperficial content.

1ſt. Of Glazoning and Maſoning.

IN glazoning and maſoning, demenſions are taken in feet and inches, and the content given in ſquare feet.

EXAMPLES.

1ſt How many feet of glazing are there in a window which is 5.5 feet long, and 3.8 broad?

5.5
3.8
440
165
20 90

Here you ſee the operation is performed decimally, which gives for anſwer 20.90 feet.

2d Suppoſe a yard 22 feet 4 inches long, and 19 feet 7 inches in breadth; to be paved with free ſtone; how many ſquare feet of payment are there in the ſame? *Anſwer* 437 *feet* 4 *inches* 4 *parts;* found by aliquot parts thus,

feet *in*
22 - 4
19 - 7
201 - 0
22 - 40
11 - 02
1 - 10 - 4
Anſwer 437 - 4 - 4

IN performing this operation, I firſt multiply my my 22 feet 4 inches, by 19 feet in this manner, viz. I ſay 9 times 4 is 36, which being 3 feet, I therefore ſet down a nought under my inches, and carry 3 feet to the line of feet, ſaying, 9 times 2 is 18 and 3 I carry is 21; and ſo on as in ſimple Multiplication to the end of that line; I next begin to multiply by my one, which being one ten, I therefore ſay 10 times 4 inches is 40, or once 4 is 4, which is the ſame thing; only remember that your 4 poſſeſs the place of tens, and paſſing on to feet, I in like manner ſay, once 2 is 2, &c. obſerving as before, to place the 1ſt figure in the 2d place of integers, which line being finiſhed, I then have 22 feet, 4 inches multiplied by 19 feet; but it yet remains to multiply 22 feet 4 inches, by 7 inches. Now it is plain that 24 feet 4 inches multiplied by 1 foot, would have ſtill remained 22 feet 4 inches, therefore I take alliquot parts, inſtead of multiplying by 7 inches, viz. for 6 inches (which is $\frac{1}{2}$ foot) I take $\frac{1}{2}$ of 22 feet 4 inches, which gives 11 feet 2 inches, and again for the 1 inch that remains, I take $\frac{1}{6}$ of 11 feet 2 inches, which ſeveral ſums being all added together give the whole content 437 feet, 4 inches, 4 parts. Or it might be done in this manner.

feet	*inches*	
22	4	
	6	
134	0	
	3	
402	0	
22	4	
11	2	
1	10	4
437	4	4

HERE according to practical Multiplication (becauſe it is tedious to multiply by 19, the Multiplicand being a mixt number) I multiply firſt by 6, and then again that product by 2, which is the ſame as if I multiplied 22 feet 4 inches by 18 feet, 3 × 6 being equal = 18. Then for the 1 foot more, I take the number given to be multiplied, viz. 22 feet 4 inches, and again take the half of that for 6 inches, and for the 1 inch, take $\frac{1}{6}$ of that line, which being all added together gives the contents 437 *feet*, 4 *inch*, 4 *parts*, as before.

Painters, Joyners, and Plaiſterers work.

IN thoſe ſeveral kinds of work, dimenſions are taken in feet and inches but the contents given in ſquare yards, found by dividing the ſuperficial feet by 9.

EXAMPLES.

1ſt. In Painting.

The height of a room being 10 feet 6 inches, and 80 feet 4 inches about; I demand the content in ſquare yards?

feet *in.*
80 : 4
10 : 6
803 : 4
40 : 2
9)843 : 6
yds. *ft.* *in.*
93 : 6 : 6 *Anſwer* 93 : 6 : 6

2d

2d. Joyner's Work.

How many yards of wainſcot doth that room take up, whoſe height is 12 ſeet 3 inches, and compaſs 104 ſeet 6 inches?

feet : *in.*
104 : 6
12 : 3

1254 : 0
26 : 1 : 6

9)1280 : 1 : 6

142 : 2 : 1 : 6

yds. *ft.* *in.* *pts.*
Anſwer 142 : 2 : 1 : 6

IN performing this queſtion, I firſt multiply 104 feet 6 inches by 12, as in compound Multiplication; then becauſe 3 inches is ¼ of a foot, I therefore take ¼ of 104 feet 6 inches, which placing below my other line, and adding together, I have 1280 feet 1 inch and 6 parts, which ſum dividing by 9 (9 ſquare feet being a ſquare yard) I have for anſwer 142 yards, 2 feet, 1 inch and 6 parts.

F I N I S.